(23) $\vec{b} = -2\vec{i} + 5\vec{j} = \langle -2, 5 \rangle$

$\hookrightarrow |\vec{b}|^2 = 2^2 + 5^2$

$= 29$

$|\vec{b}| = \sqrt{29}$

unit vector $\vec{u} = \dfrac{\vec{b}}{|\vec{b}|}$

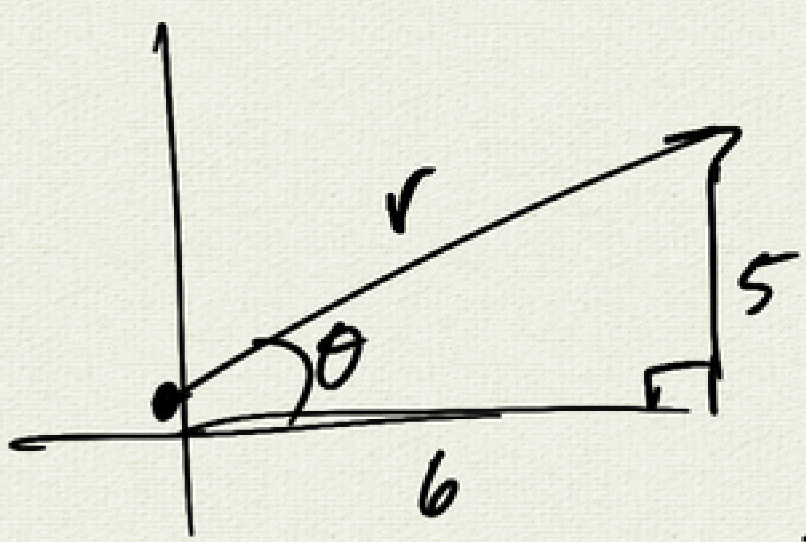
$= \dfrac{1}{\sqrt{29}} \langle -2, 5 \rangle$

$= \left\langle -\dfrac{2}{\sqrt{29}}, \dfrac{5}{\sqrt{29}} \right\rangle$

$= -\dfrac{2}{\sqrt{29}} \vec{i} + \dfrac{5}{\sqrt{29}} \vec{j}$

---

(29) $\langle 6, 5 \rangle$ magnitude & direction



$r^2 = 5^2 + 6^2$

$= 25 + 36$

$= 61$

$r = \sqrt{61}$ magnitude

$\tan\theta = \dfrac{5}{6}$

$\theta = \tan^{-1} \dfrac{5}{6}$

(check quadrant)

$y = r\sin\theta$

$5 = \sqrt{61}\,\sin\theta$

$\Rightarrow \sin\theta = \dfrac{5}{\sqrt{61}}$

$\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

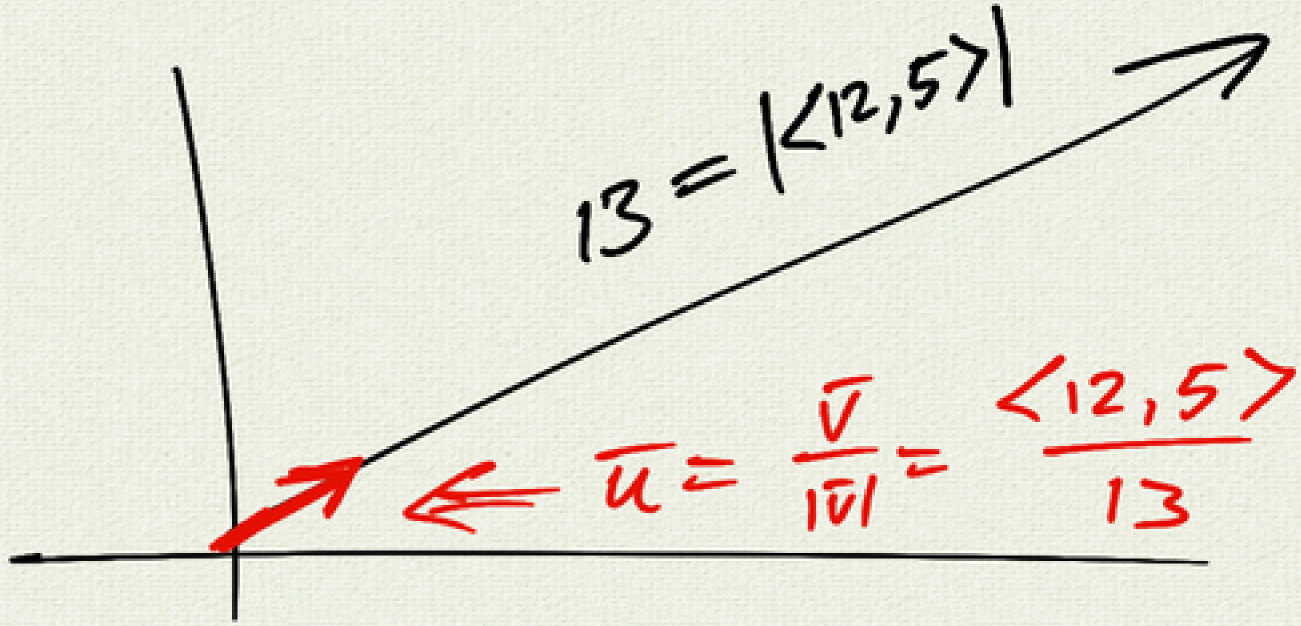
$\theta = \sin^{-1}\left(\dfrac{5}{\sqrt{61}}\right)$ exact

$\approx$

(25) $\bar{d} = -\frac{1}{3}\vec{i} + \frac{5}{2}\vec{j}$

find unit vector $\bar{u} = \frac{\bar{d}}{|\bar{d}|}$

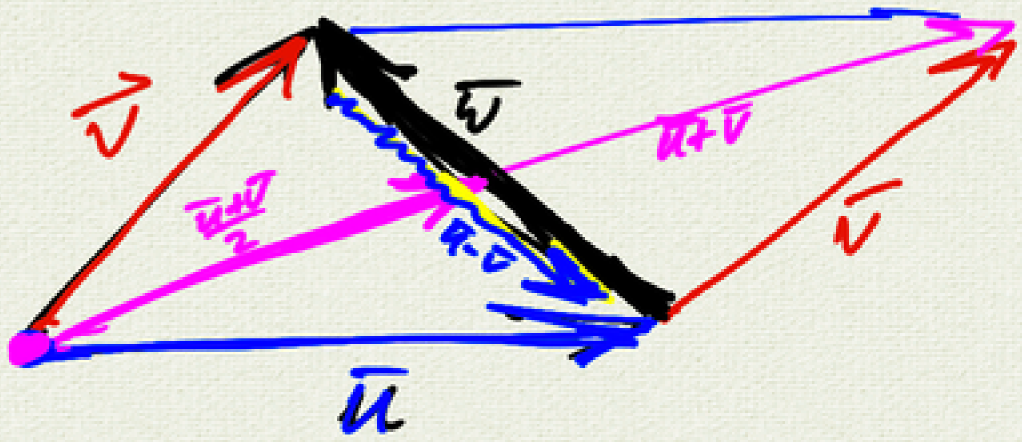
$\bar{v} = \langle 12, 5 \rangle$



$\Leftarrow \bar{u} = \frac{\bar{v}}{|\bar{v}|} = \frac{\langle 12, 5 \rangle}{13} = \boxed{\frac{1}{13}\langle 12, 5 \rangle}$

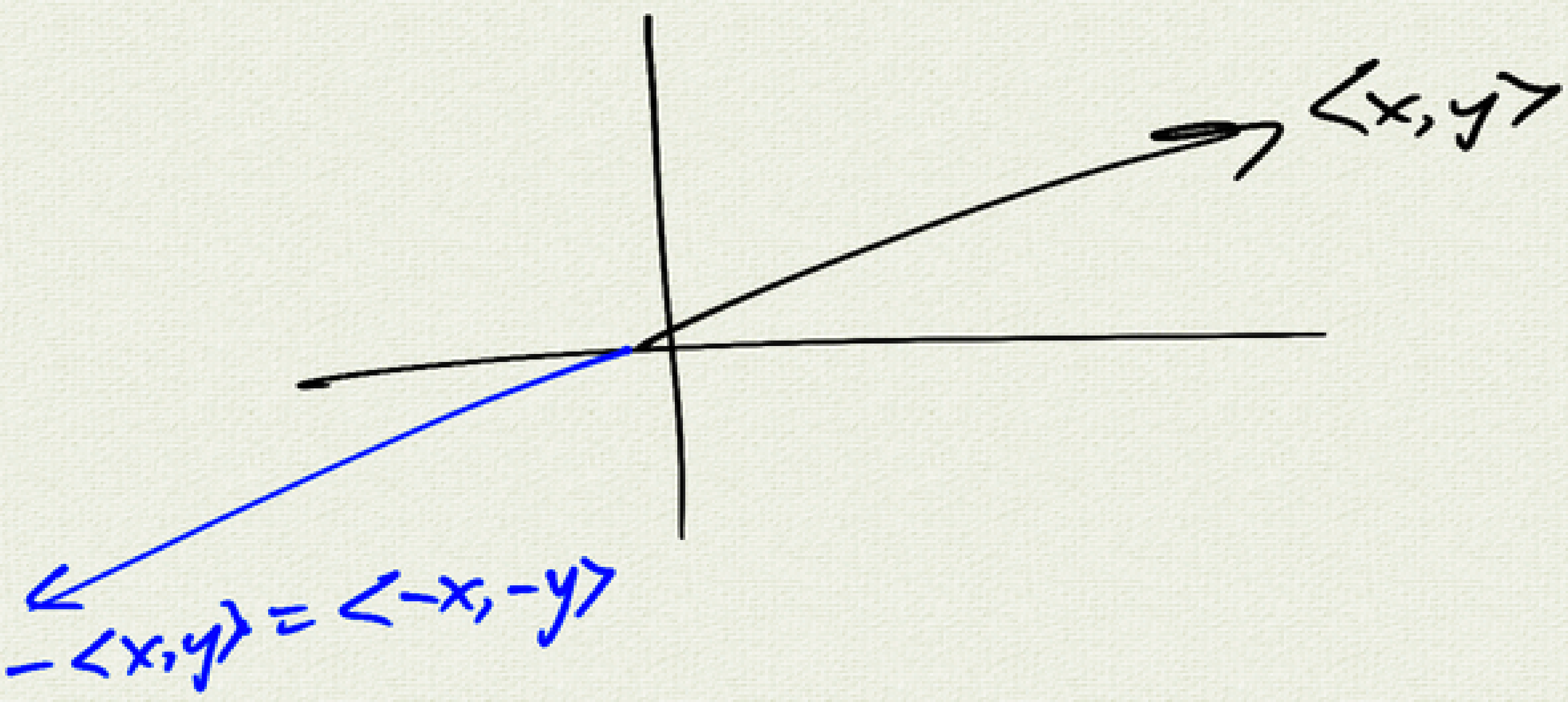
$\Rightarrow |\bar{u}| = 1$

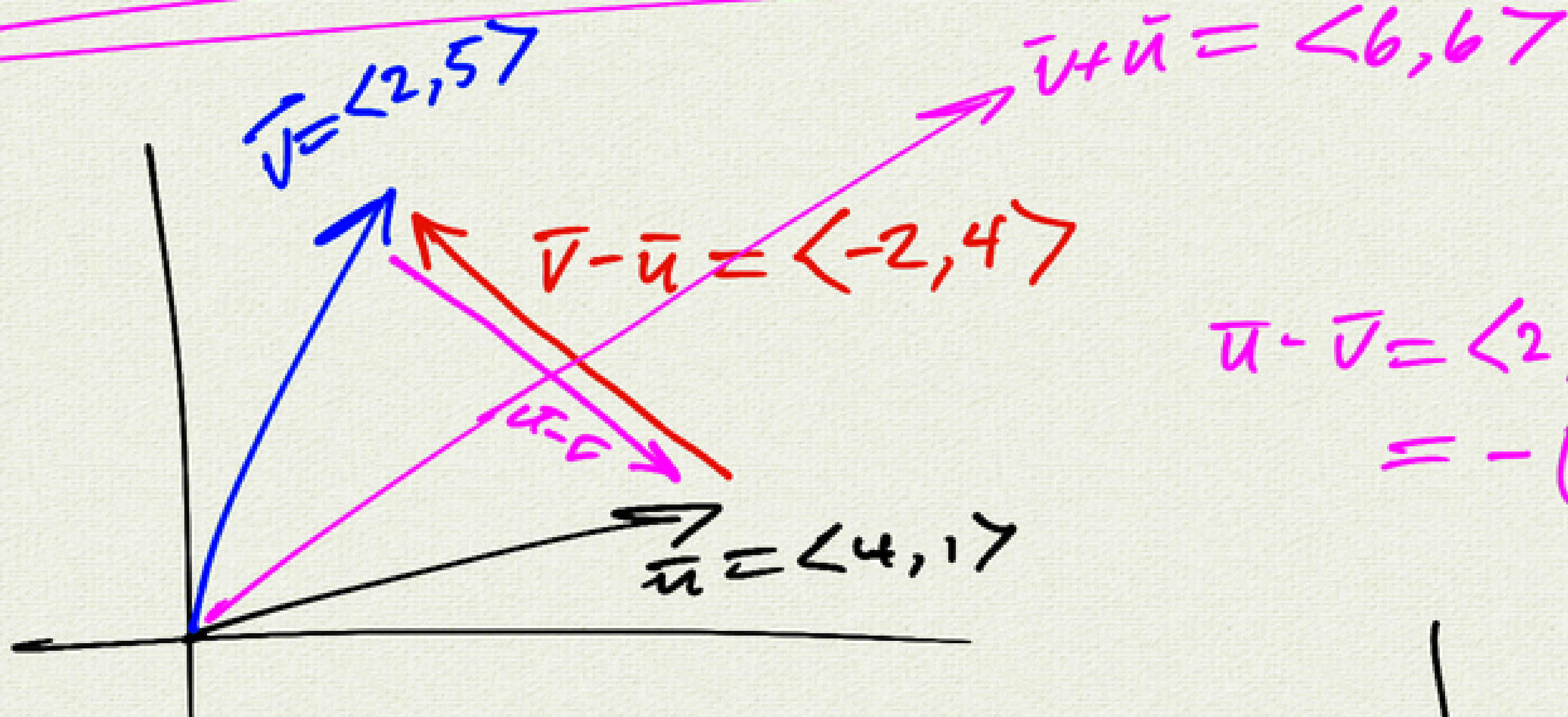
unit vector in same direction as $\langle 12, 5 \rangle$



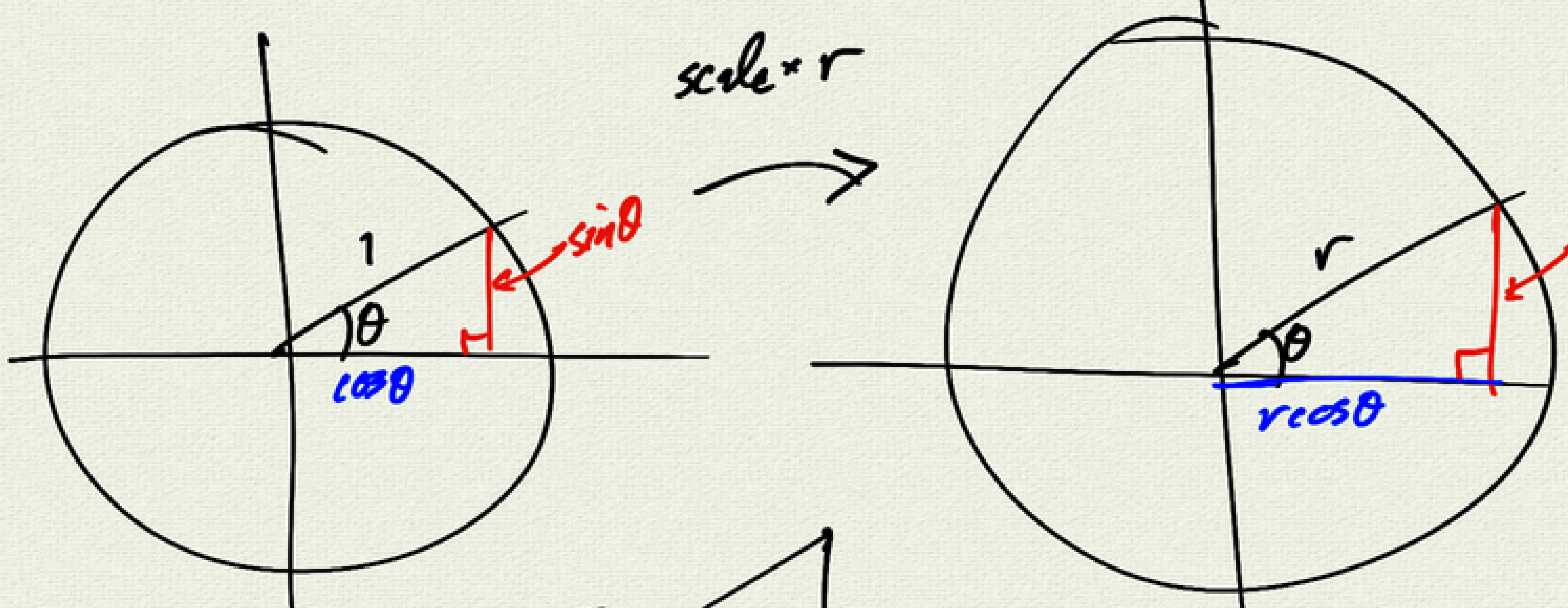
$\bar{v} = \bar{u} + \bar{w}$

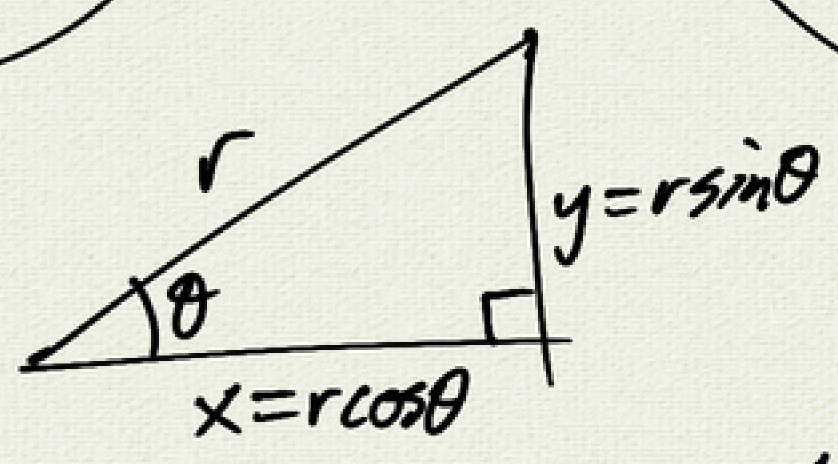
$\bar{w} = \bar{v} - \bar{u}$





$\bar{u} - \bar{v} = \langle 2, -4 \rangle$
$= -(\bar{v} - \bar{u})$





$r^2 = x^2 + y^2$

$\tan\theta = y/x$

3.2 Dot Product

$\bar{u} = \langle x_1, y_1 \rangle$ $\quad k \in \mathbb{R}$

$\bar{v} = \langle x_2, y_2 \rangle$

"is a member of"

"k is a real #"

basic operations:

addition $\quad \bar{u} + \bar{v} = \langle x_1 + x_2 , y_1 + y_2 \rangle$

scalar multiplication $\quad k\bar{u} = \langle kx_1 , ky_1 \rangle$

---

dot product (scalar product)

define $\bar{u} \cdot \bar{v} = x_1 x_2 + y_1 y_2$

example: $\langle 1,2 \rangle \cdot \langle 3,4 \rangle = 3 + 8 = 11$

vector $\quad$ vector $\quad$ not a vector

$\vec{i} \cdot \vec{i} = \langle 1,0 \rangle \cdot \langle 1,0 \rangle = 1$

$\vec{j} \cdot \vec{j} = 1$

$\vec{i} \cdot \vec{j} = \langle 1,0 \rangle \cdot \langle 0,1 \rangle = 0 + 0 = 0$

$\vec{j} \cdot \vec{i} = 0$

$\bar{u} \cdot \bar{v} = \bar{v} \cdot \bar{u}$ $\quad$ commutative

$\bar{u} = \langle x_1 , y_1 \rangle$

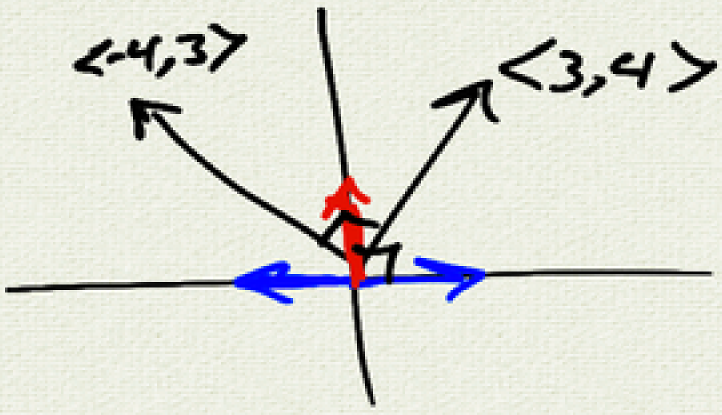
$\bar{v} = \langle x_2 , y_2 \rangle$

$\bar{u} \cdot \vec{0} = 0$

$\left.\begin{aligned} \vec{u} \cdot \vec{i} &= \langle x_1 , y_1 \rangle \cdot \langle 1,0 \rangle = x_1 \\ \bar{u} \cdot \vec{j} &= \langle x_1 , y_1 \rangle \cdot \langle 0,1 \rangle = y_1 \end{aligned}\right]$ components

$\bar{u} \cdot \vec{u} = \langle x_1 , y_1 \rangle \cdot \langle x_1 , y_1 \rangle = x_1^2 + y_1^2 = |\bar{u}|^2$

$\langle 3,4 \rangle \cdot \langle -4,3 \rangle = 0$



definition:

$\bar{u}$ and $\bar{v}$ are orthogonal if $\bar{u} \cdot \bar{v} = 0$

orthogonal

$$\boxed{\vec{u}\cdot(\vec{v}+\vec{w}) \stackrel{?}{=} \bar{u}\cdot\bar{v}+\bar{u}\cdot\bar{w}}$$

yes distributive

$$\langle x_1,y_1\rangle \cdot \left(\langle x_2,y_2\rangle + \langle x_3,y_3\rangle\right)$$

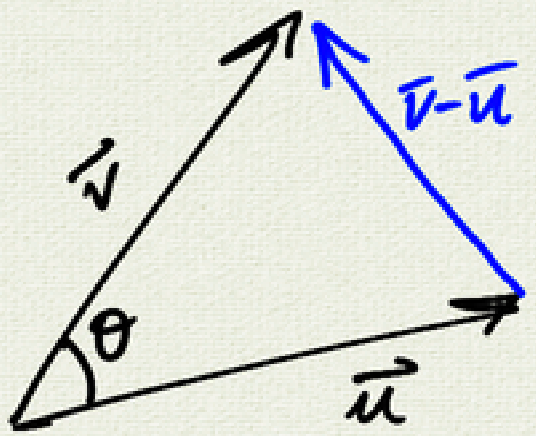
$$\stackrel{?}{=} \langle x_1,y_1\rangle\cdot\langle x_2,y_2\rangle + \langle x_1,y_1\rangle\cdot\langle x_3,y_3\rangle$$

---

distributive $\Longrightarrow$ FOIL

$$(\bar{u}+\bar{v})\cdot(\bar{w}+\bar{z}) = \underset{F}{\bar{u}\cdot\bar{w}} + \underset{O}{\bar{u}\cdot\bar{z}} + \underset{I}{\bar{v}\cdot\bar{v}} + \underset{L}{\bar{v}\cdot\bar{z}}$$

$$\bar{u}+(\bar{v}-\bar{u}) = \bar{v}$$



$$|\bar{u}|^2 = \bar{u}\cdot\bar{u}$$

$$|\bar{v}-\bar{u}|^2 = (\bar{v}-\bar{u})\cdot(\bar{v}-\bar{u})$$

$$= \bar{v}\cdot\bar{v} - \bar{v}\cdot\bar{u} - \bar{u}\cdot\bar{v} + \bar{u}\cdot\bar{u}$$

$$|\bar{v}-\bar{u}|^2 = |\bar{u}|^2 + |\bar{v}|^2 - 2\boxed{\bar{u}\cdot\bar{v}}$$

$$|\bar{v}-\bar{u}|^2 = |\bar{u}|^2 + |\bar{v}|^2 - 2\boxed{|\bar{u}||\bar{v}|\cos\theta}$$

Law of Cosines

$$\boxed{\bar{u}\cdot\bar{v} = |\bar{u}||\bar{v}|\cos\theta}$$

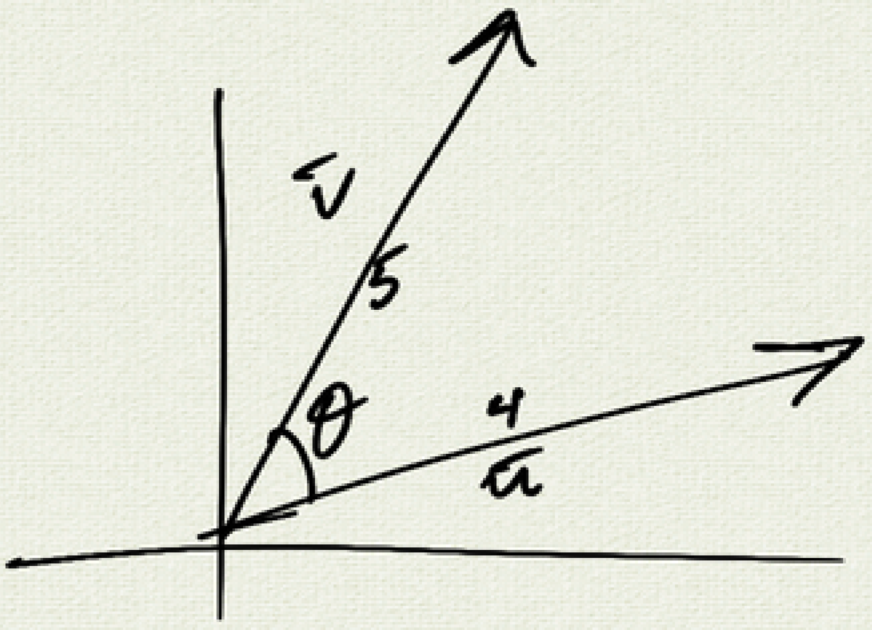
alternate definition of dot product

$$\Longrightarrow \cos\theta = \frac{\bar{u}\cdot\bar{v}}{|\bar{u}||\bar{v}|} \Longleftarrow$$

find angle θ between two vectors

example: $\bar{u} = \boxed{4\left\langle \frac{\sqrt{3}}{2}, \frac{1}{2} \right\rangle} = \langle 2\sqrt{3}, 2 \rangle$

$\bar{v} = 5\left\langle \frac{1}{2}, \frac{\sqrt{3}}{2} \right\rangle = \left\langle \frac{5}{2}, \frac{5\sqrt{3}}{2} \right\rangle$ given



find angle θ between $\bar{u}$ and $\bar{v}$

$$\cos\theta = \frac{\bar{u}\cdot\bar{v}}{|\bar{u}||\bar{v}|}$$

4 5

$|\bar{u}| = |\langle 2\sqrt{3}, 2 \rangle|$

$|\bar{u}|^2 = (2\sqrt{3})^2 + 2^2$

$= 12 + 4$

$= 16$

$|\bar{u}| = 4$

$\bar{u}\cdot\bar{v} = \langle 2\sqrt{3}, 2 \rangle \cdot \left\langle \frac{5}{2}, \frac{5\sqrt{3}}{2} \right\rangle$

$= 5\sqrt{3} + 5\sqrt{3}$

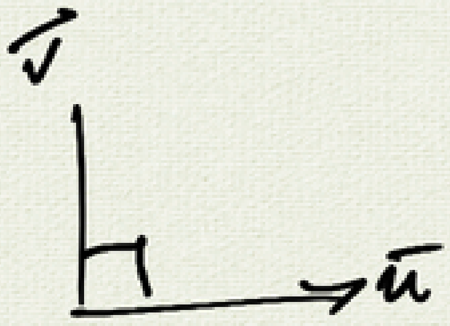
$= 10\sqrt{3}$

$\Rightarrow \cos\theta = \frac{10\sqrt{3}}{4\cdot 5} = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = \frac{\pi}{6}$

---

orthogonality: $\bar{u}\cdot\bar{v} = 0 \Longleftrightarrow \bar{u}, \bar{v}$ orthogonal



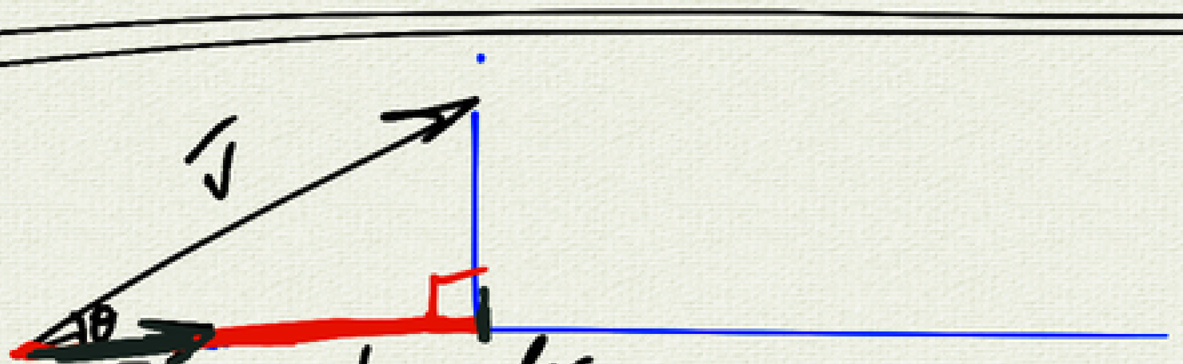
$\bar{u}\cdot\bar{v} = |\bar{u}||\bar{v}|\cos\theta = 0$

$|\bar{u}| = 0$ or

$|\bar{v}| = 0$ or

$\cos\theta = 0 \longrightarrow \theta = \frac{\pi}{2}$ perpendicular

$\bar{u} = \langle 0, 0 \rangle = \bar{0}$



unit vector

$|\bar{u}| = 1$

$v\cos\theta$

$|\bar{v}|\cos\theta$ = length of shadow along $\bar{u}$

= projection