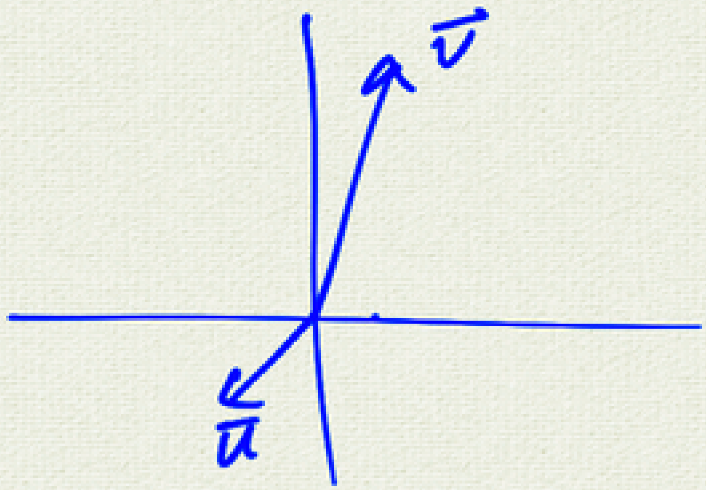
(33) $\vec{u} = \langle -1, -1 \rangle$

$\vec{v} = \langle 1, 5 \rangle$



$\vec{u} \cdot \vec{v} = -1 - 5 = -6$

$|\vec{u}| = \sqrt{2}$

$|\vec{v}| = \sqrt{26}$

$\Longrightarrow \cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|} = \dfrac{-6}{\sqrt{2}\sqrt{26}}$

$\theta \approx \boxed{\cos^{-1}\left(\dfrac{-6}{2\sqrt{13}}\right)} \approx 2.55$

exact

$\approx \underline{146.3^\circ}$

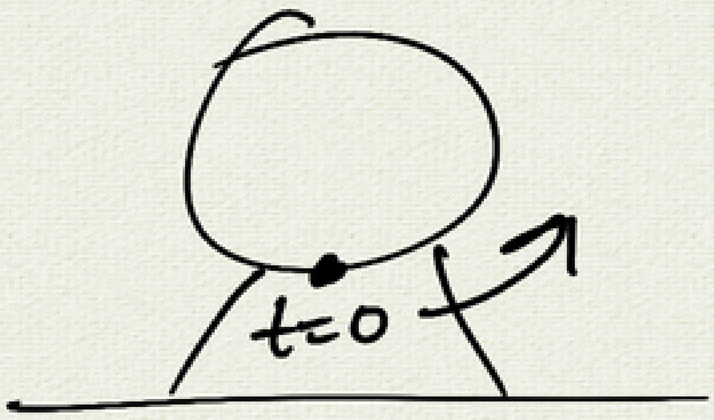
approx

$\vec{u} = \langle x_1, y_1 \rangle$
$\vec{v} = \langle x_2, y_2 \rangle$ $\Big\}$ $\vec{u} \cdot \vec{v} = x_1 x_2 + y_1 y_2$

$\boxed{\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}|\cos\theta}$

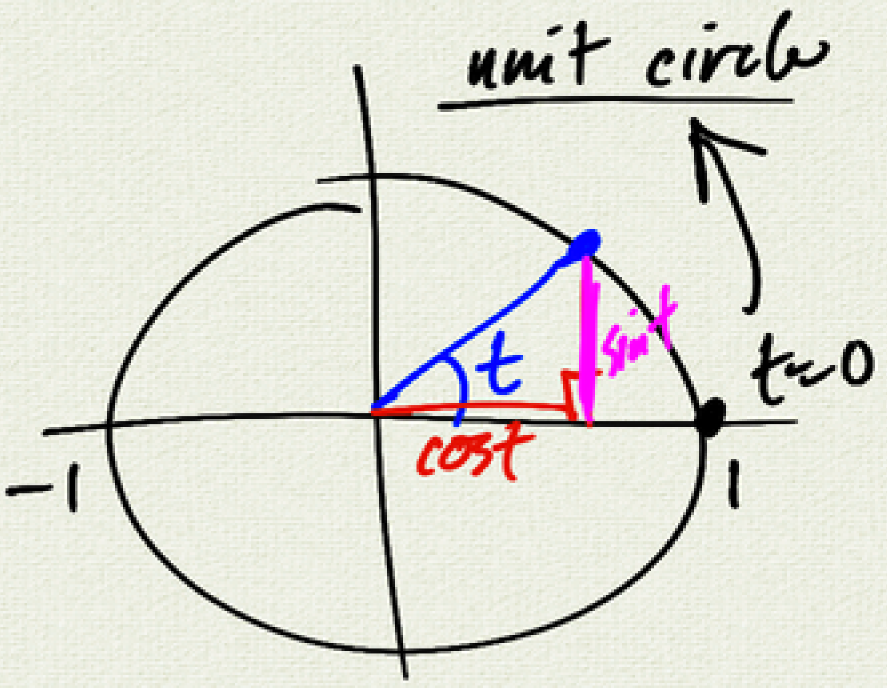
$\Longrightarrow \cos\theta = \dfrac{\vec{u} \cdot \vec{v}}{|\vec{u}||\vec{v}|}$

3.3 Parametric Equations



$x(t) =$
$y(t) =$ } parametric equations

t — parameter

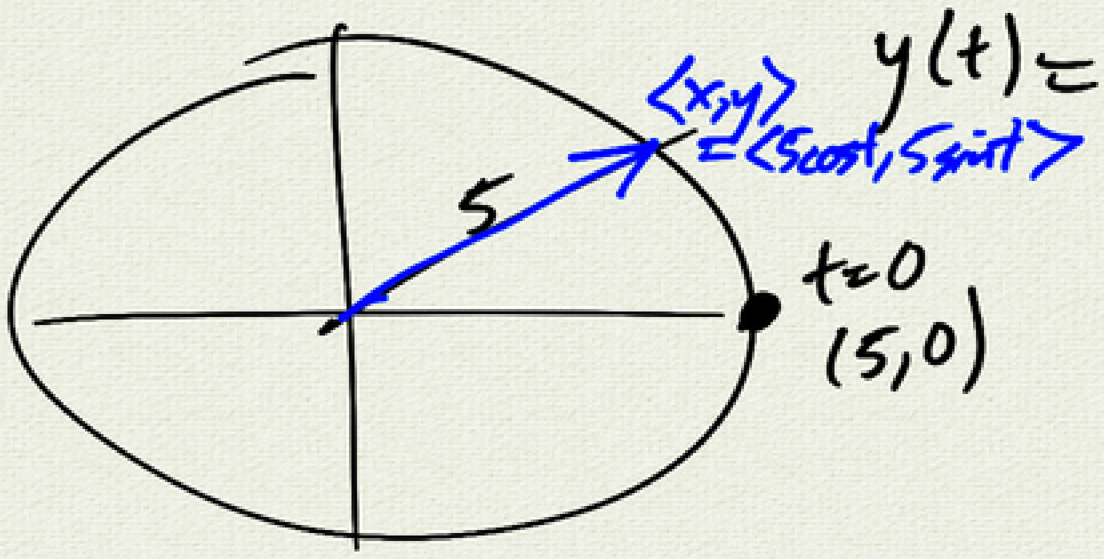


$x(t) = \cos t$
$y(t) = \sin t$

t	$x(t)$	$y(t)$
0	1	0
$\pi/2$	0	1
π	-1	0
$3\pi/2$	0	-1
2π	1	0

---

radius 5



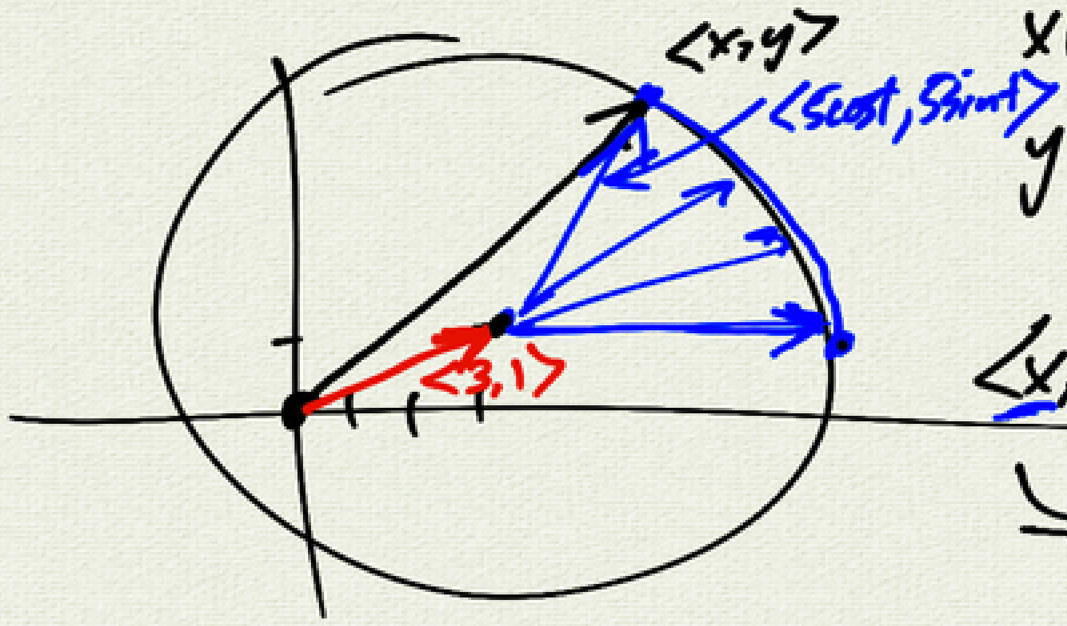
$x(t) = 5\cos t$
$y(t) = 5\sin t$

$\langle x, y\rangle = \langle 5\cos t, 5\sin t\rangle$

---

circle radius 5, center (3, 1)

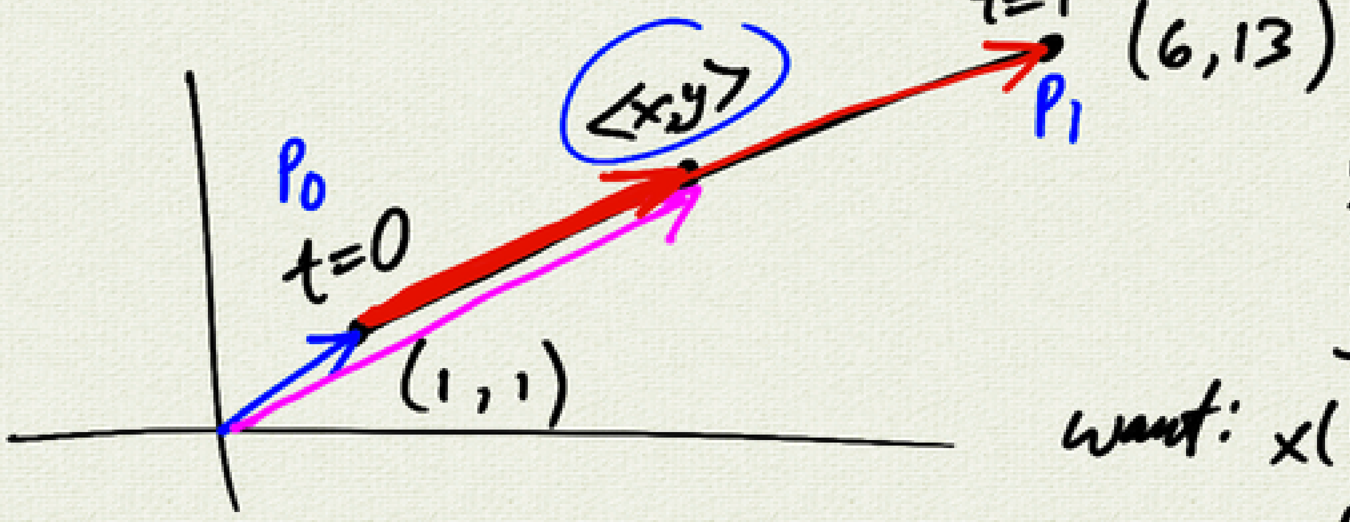
$\cos bt \Rightarrow$ period $\frac{2\pi}{b}$



$x(t) = 3 + 5\cos t$
$y(t) = 1 + 5\sin t$

$\langle x,y\rangle = \langle 3,1\rangle + \langle 5\cos t, 5\sin t\rangle$

$\Longrightarrow$ $x = 3 + 5\cos t$
$y = 1 + 5\sin t$



$x(t)$

$y(t)$

want: $x(0)=1\ ,\ y(0)=1$

$x(1)=6\ ,\ y(1)=13$

Eli suggest:

$x(t)=1+5t$

$y(t)=1+12t$

$$\langle x,y\rangle = \boxed{\langle 1,1\rangle + \boxed{t\underbrace{\langle 5,12\rangle}_{\overline{P_0P_1}}}}$$

$(1,1) \longrightarrow (6,13)$

$x_1\ y_1 \qquad x_2\ y_2$

$P_1 \qquad P_2$

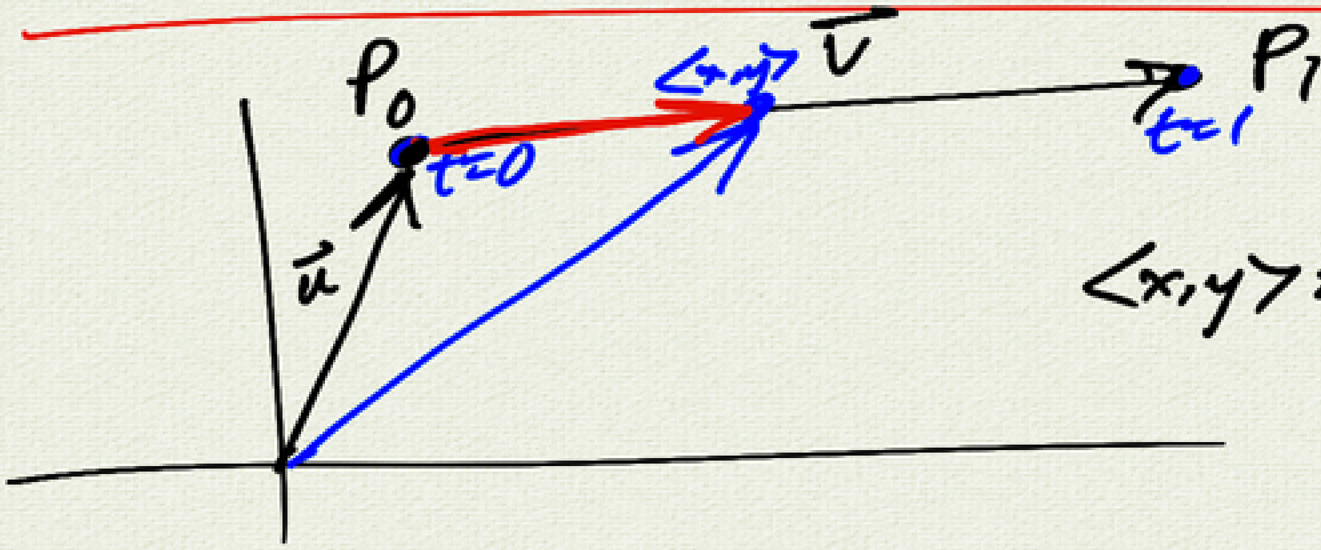
$\overline{P_1P_2} = \langle x_2-x_1, y_2-y_1\rangle$

$\overline{P_0P_1} = \langle 6-1, 13-1\rangle$

$= \langle 5,12\rangle$

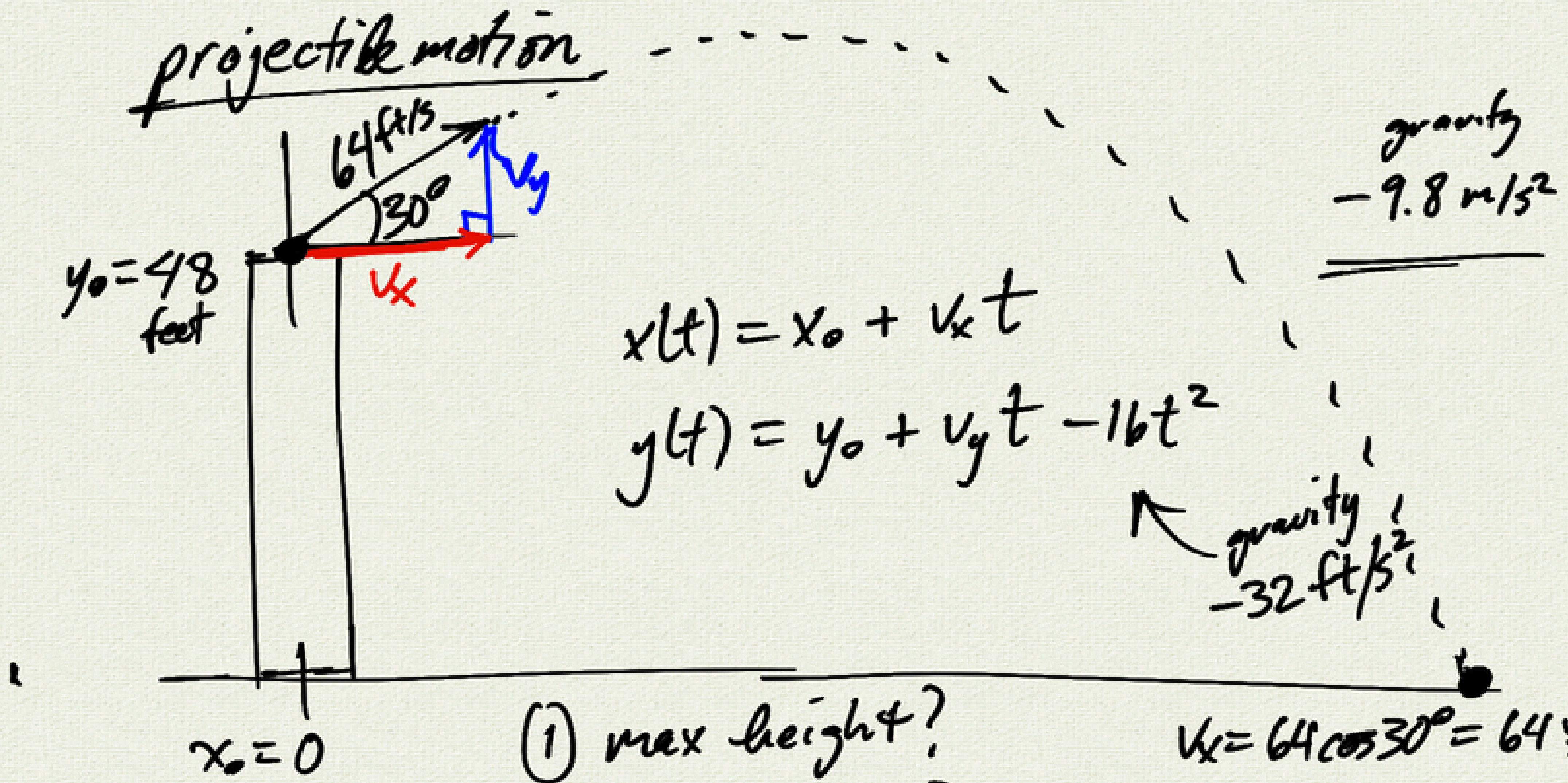
$\Rightarrow x = 1+5t$

$y = 1+12t$



$\langle x,y\rangle = \vec{u} + t\,\vec{v}$

projectile motion



$$x(t) = x_0 + v_x t$$
$$y(t) = y_0 + v_y t - 16t^2$$

gravity $-32\ ft/s^2$

(1) max height?
(2) max distance?

$v_x = 64\cos 30° = 64\frac{\sqrt{3}}{2} = 32\sqrt{3}$
$v_y = 64\sin 30 = 64 \cdot \frac{1}{2} = 32$

$$x(t) = v_x t = 32\sqrt{3}t$$
$$y(t) = y_0 + v_y t - 16t^2$$
$$= 48 + 32t - 16t^2 \Leftarrow \text{parabola}$$

---

(1) max height
at $t = \frac{-b}{2a}$
$= \frac{-32}{-32} = 1$

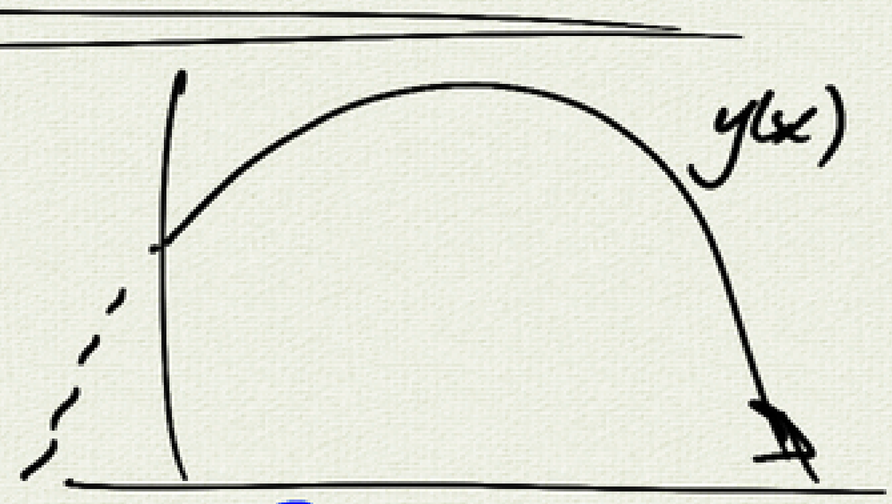
max height $y(1) = 48 + 32 - 16$
$= 64$

$0 = 48 + 32t \underline{-16}t^2$ ← a
quadratic formula
$at^2 + bt + c = 0$
$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
$= \frac{-b}{2a} \pm \boxed{}$

(2) max distance
(a) find t when $y = 0$
$$-16t^2 + 32t + 48 = 0$$
$$-16(t^2 - 2t - 3) = 0$$
$$-16(t-3)(t+1) = 0$$
$$\Longrightarrow t = -1, \boxed{3}$$

(b) find $x(3)$
$$x(3) = 32\sqrt{3}(3)$$
$$= 96\sqrt{3}$$

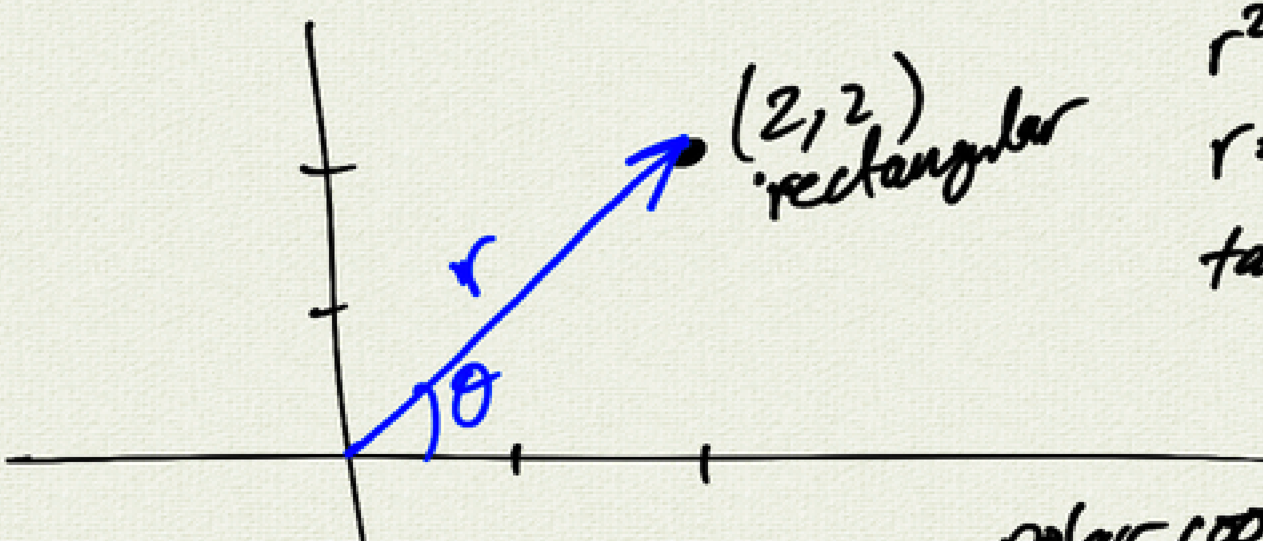
$\boxed{x(t) = 32\sqrt{3}\, t}$
$y(t) = 48 + 32t - 16t^2$



$\boxed{x = 32\sqrt{3}\, t}$
$t = \frac{x}{32\sqrt{3}} \Rightarrow y = 48 + 32t - 16t^2$
$= 48 + 32\left(\frac{x}{32\sqrt{3}}\right) - 16\left(\frac{x}{32\sqrt{3}}\right)^2$
$= 48 + \frac{1}{\sqrt{3}}x - (\ \ )x^2$ parabola

eliminate parameter t

3.4 Polar coordinates



$r^2 = x^2 + y^2$

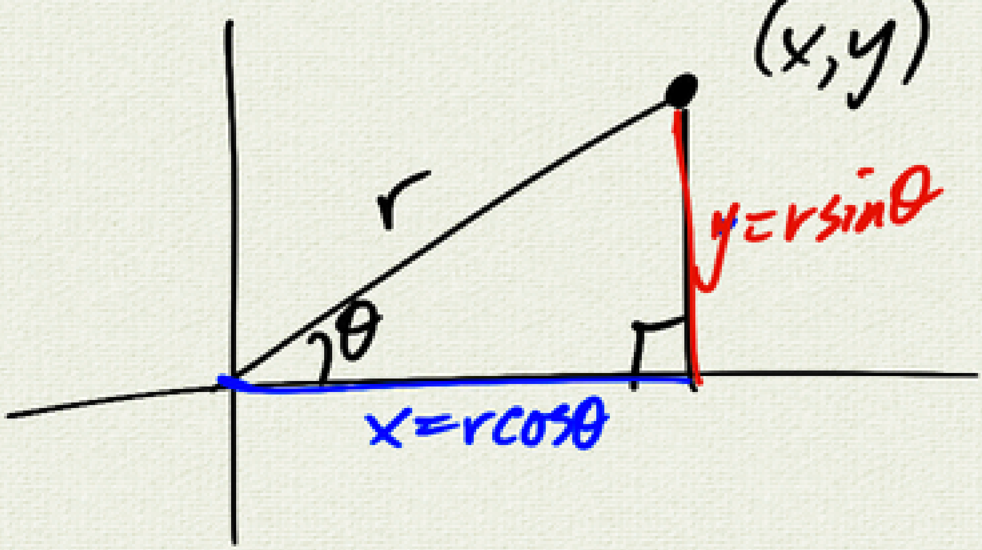
$r = \sqrt{4+4} = 2\sqrt{2}$

$\tan\theta = \frac{2}{2} = 1$

$\theta = \pi/4$

polar coordinates

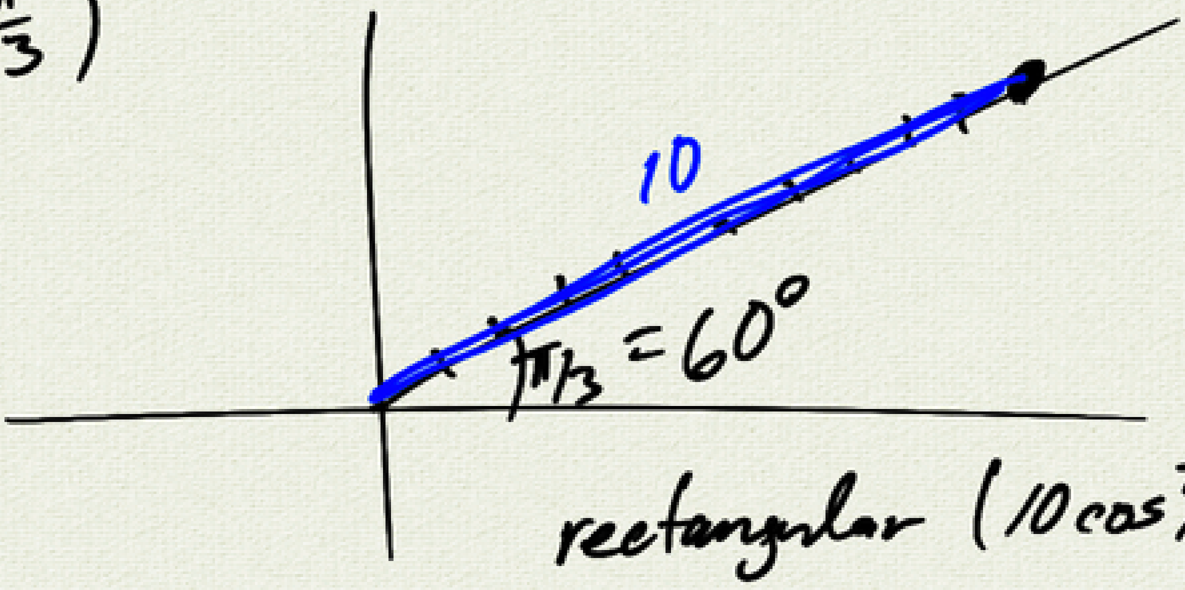
$(r, \theta) = (2\sqrt{2}, \pi/4)$



$r^2 = x^2 + y^2$

$\tan\theta = y/x$

plot polar $\left(10, \frac{\pi}{3}\right)$



rectangular $(10\cos\pi/3,\ 10\sin\pi/3)$

$= (5, 5\sqrt{3})$

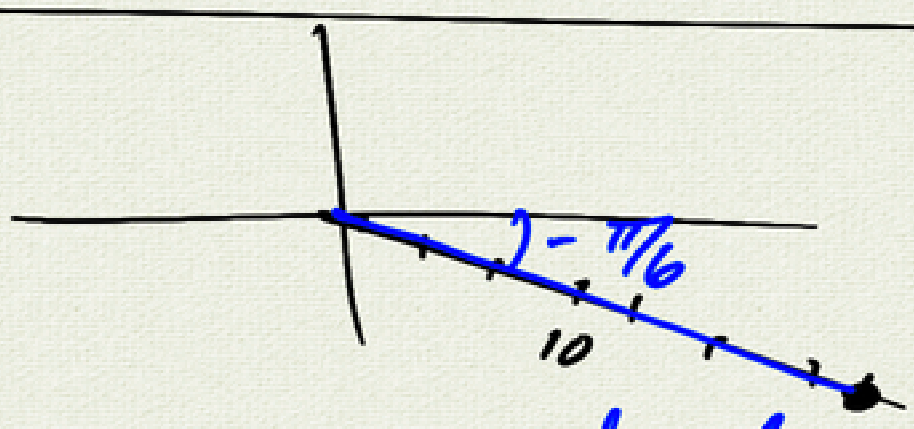
$\quad\ x \quad\ y$

---

polar

$(10, -\pi/6)$

$\ r \qquad \theta$



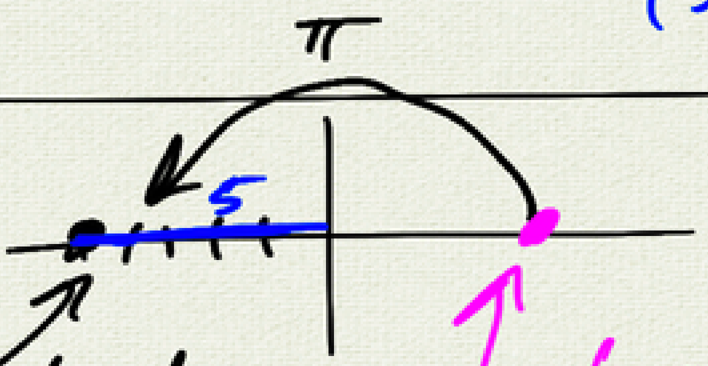
rectangular

$(5\sqrt{3}, -5)$

---

polar

$(5, \pi)$



rectangular

$(-5, 0)$

rectangular

$(5, 0)$

polar

$(-5, \pi)$

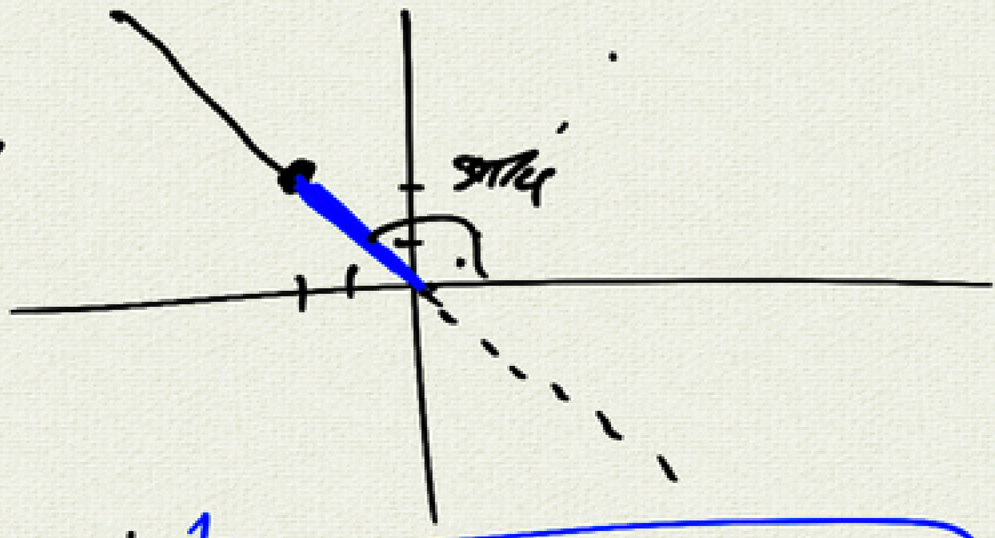
$(5, 2\pi)$

$(5, 0)$

---

rectangular $(-2, 2)$

find all polar coordinates



polar $\left(2\sqrt{2}, \frac{3\pi}{4}\right)$

$\left(-2\sqrt{2}, -\frac{\pi}{4}\right)$

$\left(2\sqrt{2}, -5\frac{\pi}{4}\right)$

$$\boxed{\begin{array}{l}\left(2\sqrt{2},\ \frac{3\pi}{4} + 2\pi k\right) \\ \left(-2\sqrt{2},\ -\frac{\pi}{4} + 2\pi k\right)\end{array}}$$

$k \in \mathbb{Z}$