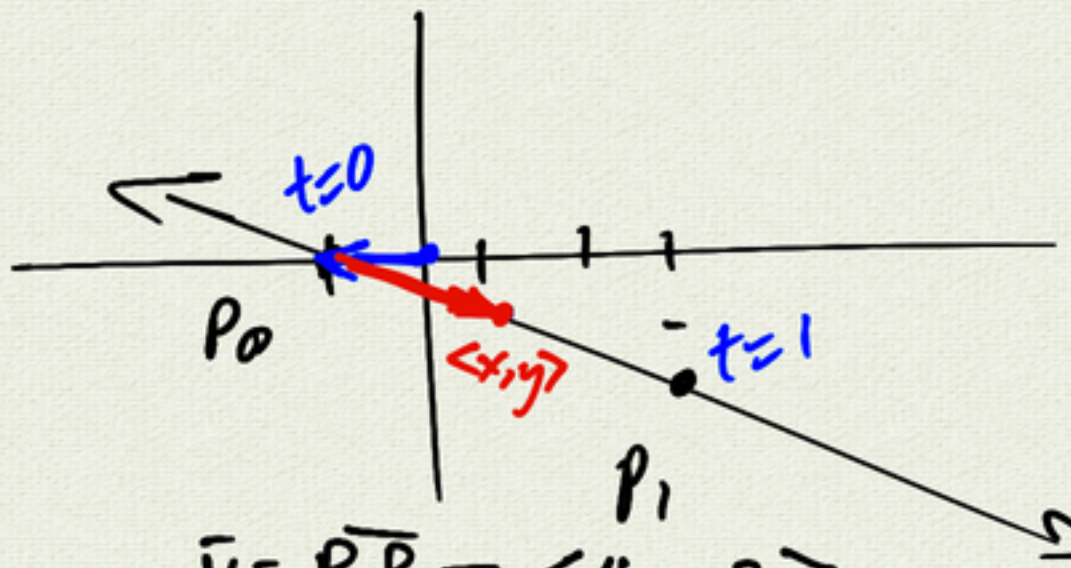
(39) Parametrize line P_1 $(3,-2)$ to P_0 $(-1,0)$

P_0 $t=0$ $\quad$ $t=1$



$\bar{v} = \overrightarrow{P_0P_1} = \langle 4, -2 \rangle$

$P_0: (-1, 0)$

$P_1: (3, -2)$

$3-(-1) = 4$ $\quad$ $-2-0 = -2$

$\langle x, y \rangle = \langle -1, 0 \rangle + t\langle 4, -2 \rangle$

$x(t) = -1 + 4t$

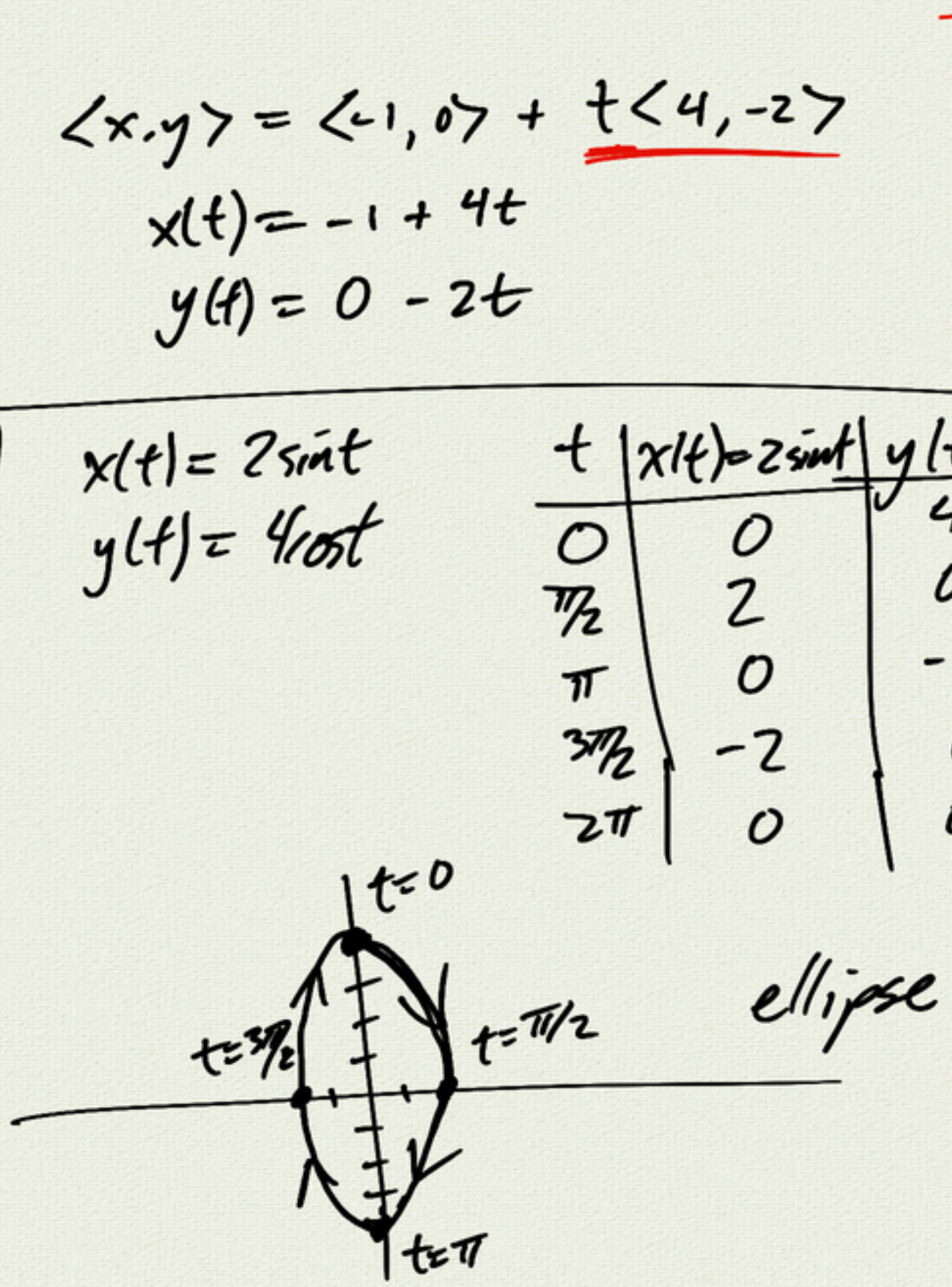
$y(t) = 0 - 2t$

---

(17) $x(t) = 2\sin t$

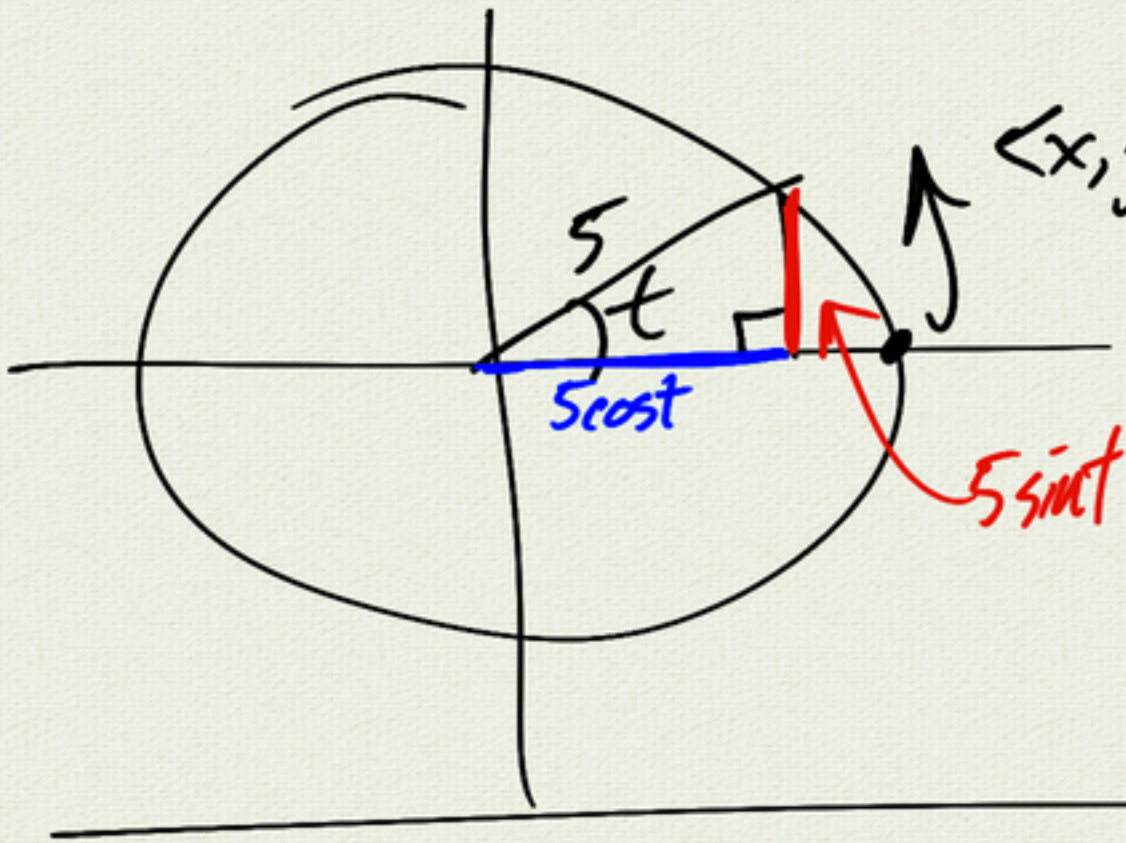
$y(t) = 4\cos t$

t	$x(t)=2\sin t$	$y(t)=4\cos t$
0	0	4
$\pi/2$	2	0
π	0	-4
$3\pi/2$	-2	0
2π	0	4



ellipse

(45)



$\langle x, y\rangle = \langle 5\cos t, 5\sin t\rangle$

$x(t) = 5\cos t$

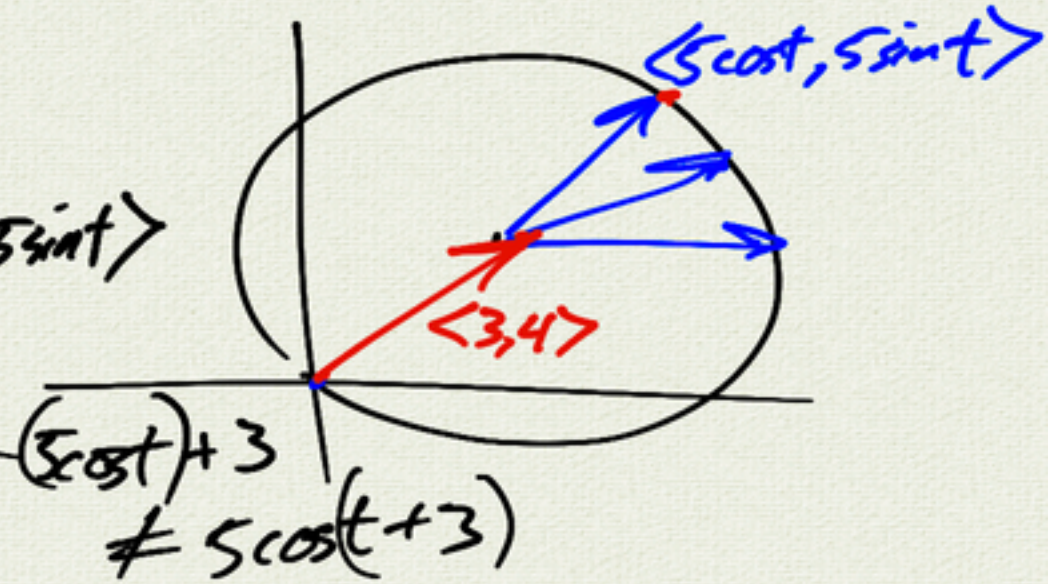
$y(t) = 5\sin t$

center $(3,4)$:

$\langle x, y\rangle = \langle 3,4\rangle + \langle 5\cos t, 5\sin t\rangle$

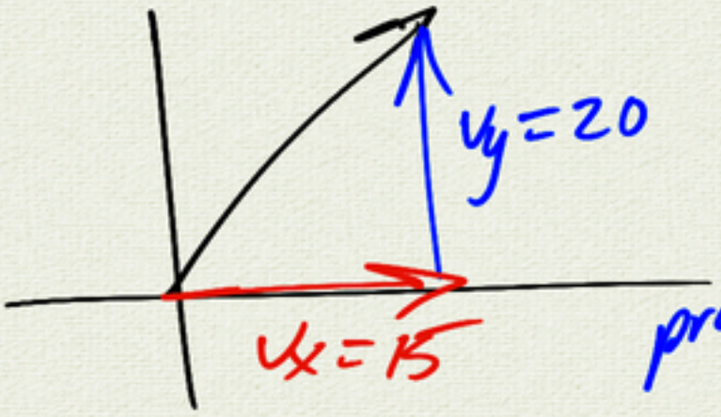
$x(t) = 3 + 5\cos t$

$y(t) = 4 + 5\sin t$



$\langle x, y\rangle = \langle 3,4\rangle + 5\langle \cos t, \sin t\rangle$

(63)



projectiles

$$y(t) = -16t^2 + 20t$$

$$(= -16t^2 + v_y t + y_0)$$

initial y velocity

initial y position

$$x(t) = v_x t + x_0$$

$$= 15t$$

---

eliminate time $\Rightarrow$ (find y as function of x)

$$x = 15t$$

$$t = x/15$$

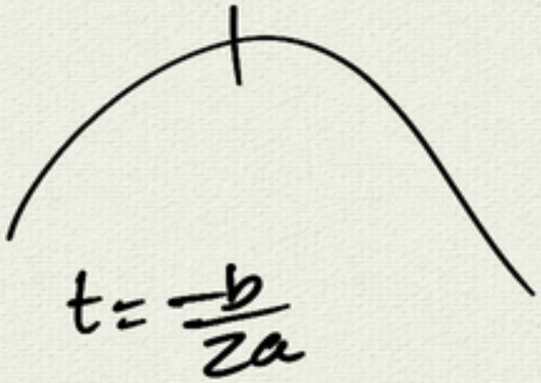
$$y = -16t^2 + 20t$$

$$= -16\left(\frac{x}{15}\right)^2 + 20\left(\frac{x}{15}\right)$$

$\Leftarrow$ $y(x)$ is a parabola

---

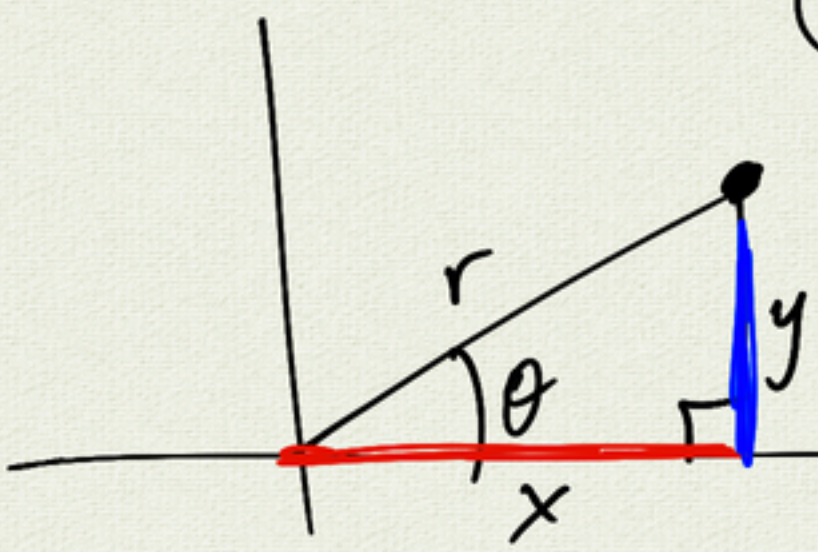
max height: find vertex of $y(t)$



$$t = \frac{-b}{2a}$$

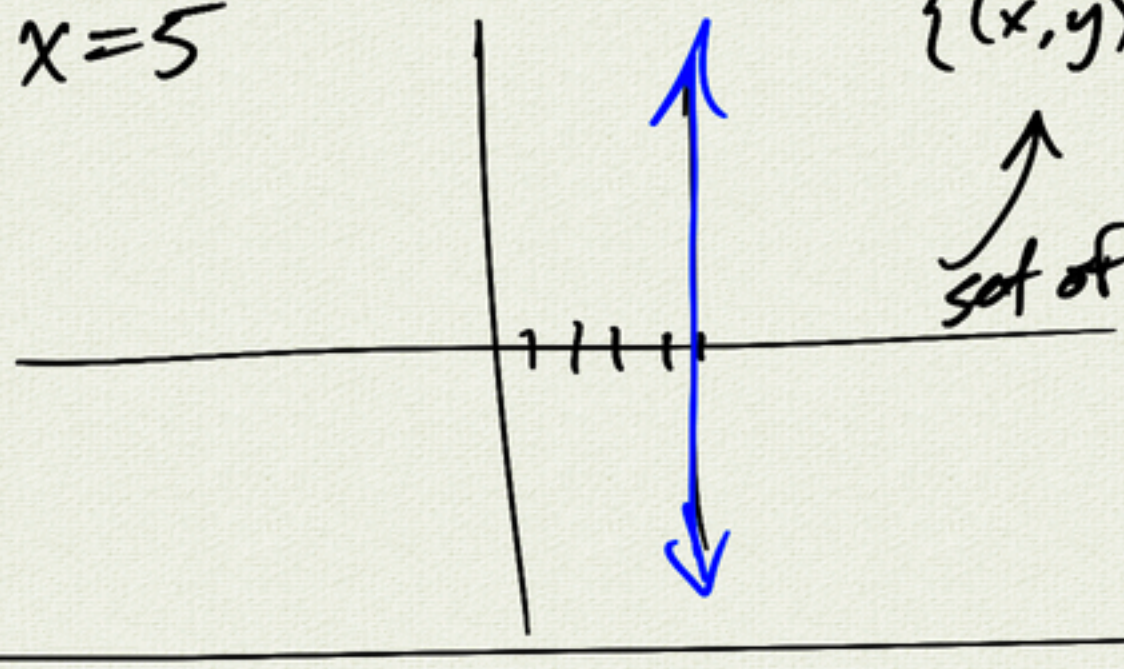
3.5 Polar Graphs

(r,θ) polar
(x,y) rectangular



$x = r\cos\theta$ $\quad r^2 = x^2 + y^2$
$y = r\sin\theta$ $\quad \tan\theta = y/x$

$x = 5$

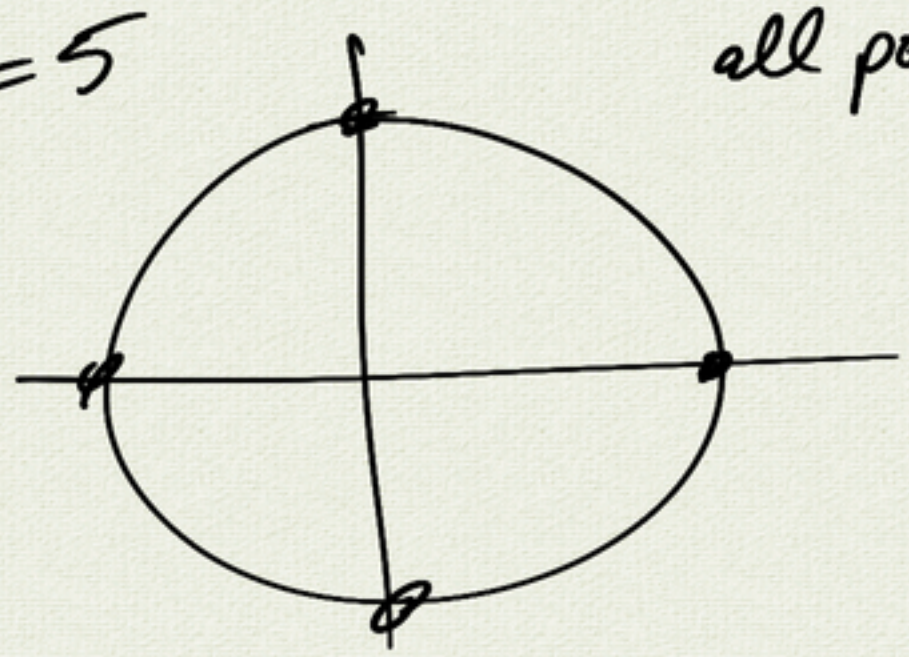


$\{(x,y) \mid x = 5\}$

"such that"

set of all (x,y)

$r = 5$



all points (r,θ)
such that $r = 5$

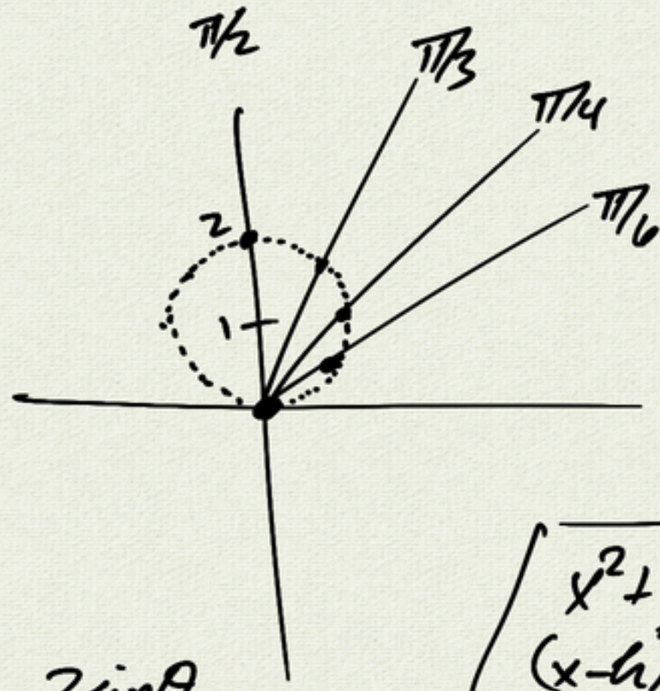
---

or: $r =$ function of θ

θ	$r = 5$
0	5
$\pi/2$	5
π	5
$3\pi/2$	5
2π	5

$r = 2\sin\theta$

θ	$r = 2\sin\theta$
0	0
$\pi/6$	$2 \cdot \frac{1}{2} = 1$
$\pi/4$	$2 \cdot \frac{\sqrt{2}}{2} = \sqrt{2} \approx 1.414$
$\pi/3$	$2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3} \approx 1.732$
$\pi/2$	$2 \cdot 1 = 2$
$2\pi/3$	$\sqrt{3}$



$r = 2\sin\theta$

$r^2 = 2r\sin\theta$

$x^2+y^2 \quad y$

$x^2 + y^2 = 2y$

$x^2 + (y^2 - 2y) = 0$

$x^2 + (y^2 - 2y + 1) = 1$

$x^2 + (y-1)^2 = 1$

circle.
radius 1
center $(0,1)$

$x^2 + y^2 = r^2$

$(x-h)^2 + (y-k)^2 = r^2$

$x = r\cos\theta$

$y = r\sin\theta$ ⟸

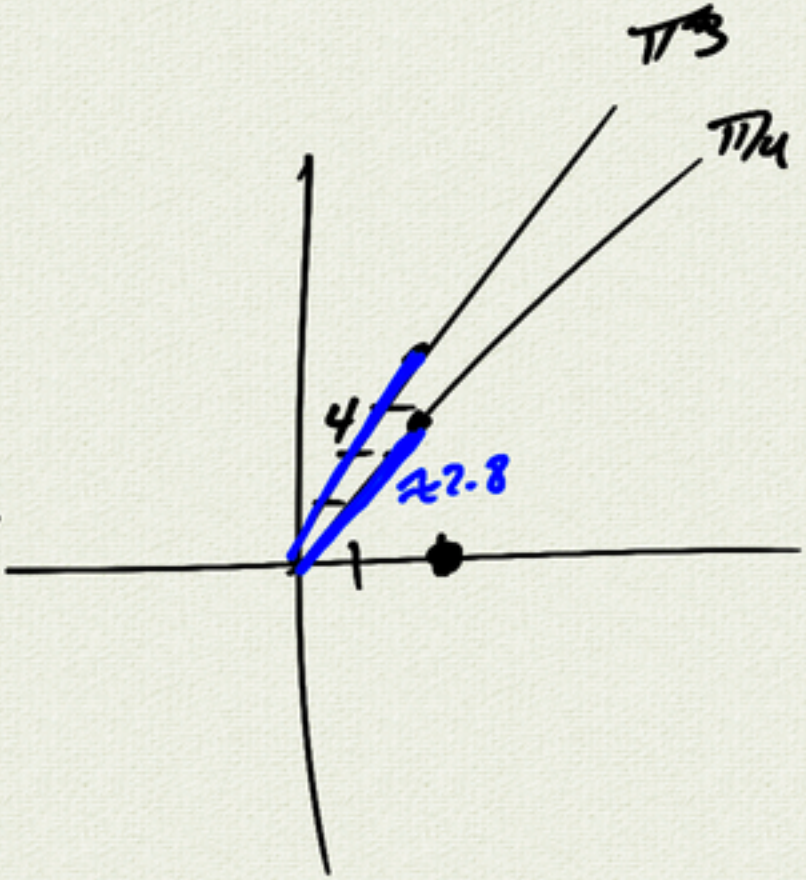
$x^2 + y^2 = r^2$

$(y-k)^2 = y^2 - 2yk + k^2$

$-2ky \quad +k^2$

$r = 2\sec\theta$

θ	$r = 2\sec\theta = \frac{2}{\cos\theta}$
0	2
$\pi/6$	$2 \cdot \frac{2}{\sqrt{3}}$
$\pi/4$	$2 \cdot \frac{2}{\sqrt{2}} = 2\sqrt{2} \approx 2.8$
$\pi/3$	$2 \cdot 2$
$\pi/2$	undefined



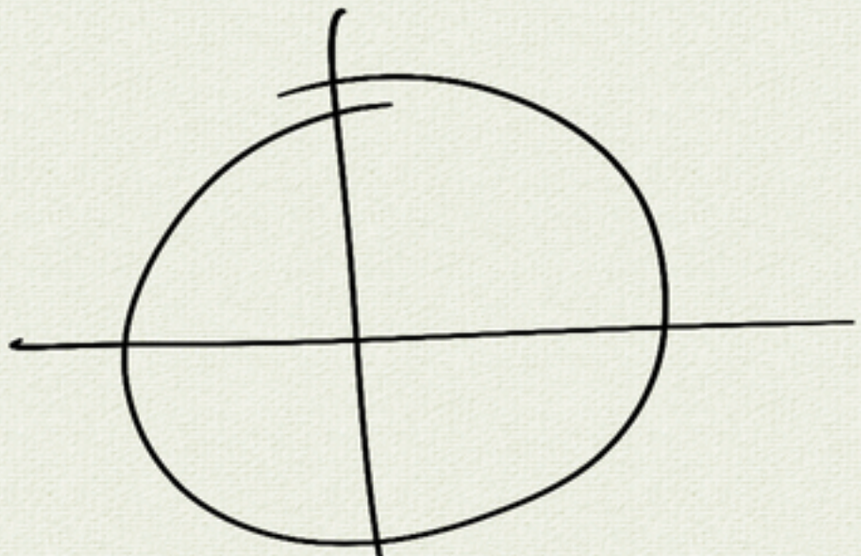
$r = 2\sec\theta$

$r = \frac{2}{\cos\theta} \Rightarrow r\cos\theta = 2$

$x = 2$

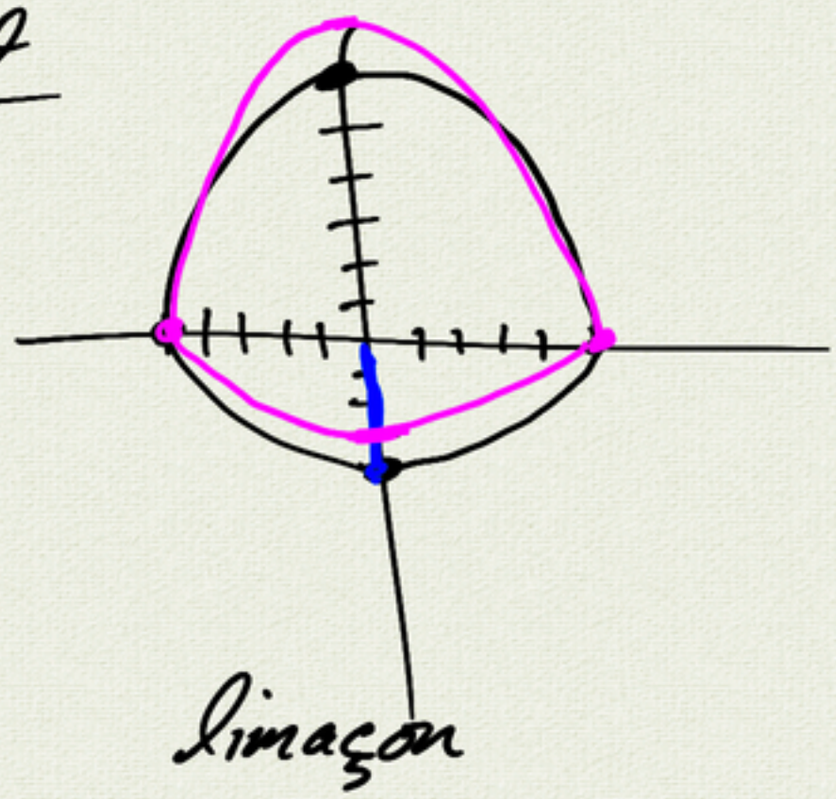
$r = 5$

θ	$r=5$
0	5
$\pi/2$	5
π	5
$3\pi/2$	5
2π	5



$r = 5 + \sin\theta$

θ	$\sin\theta$	$r=5+\sin\theta$
0	0	5
$\pi/2$	1	6
π	0	5
$3\pi/2$	-1	4
2π	0	5

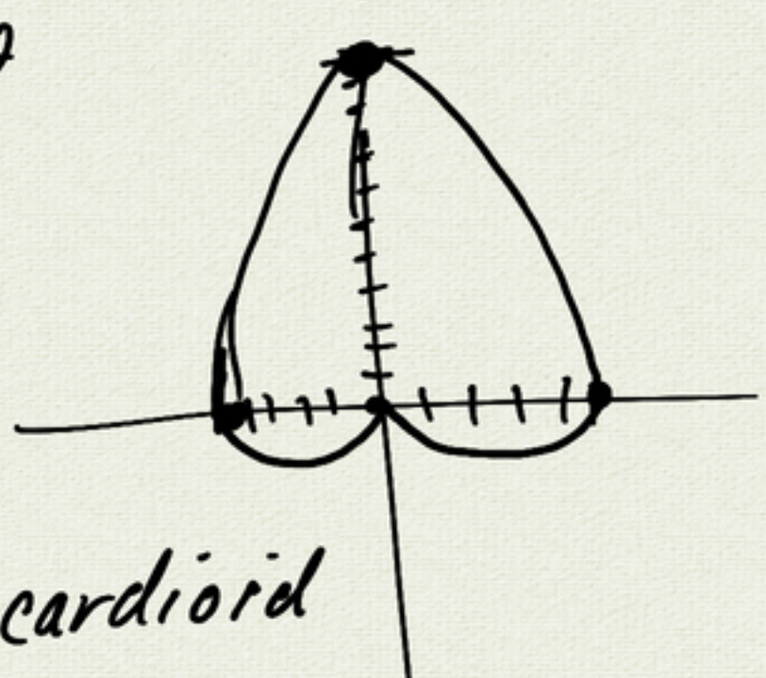


limaçon

$r = 5 + 2\sin\theta$

$r = 5 + 5\sin\theta$

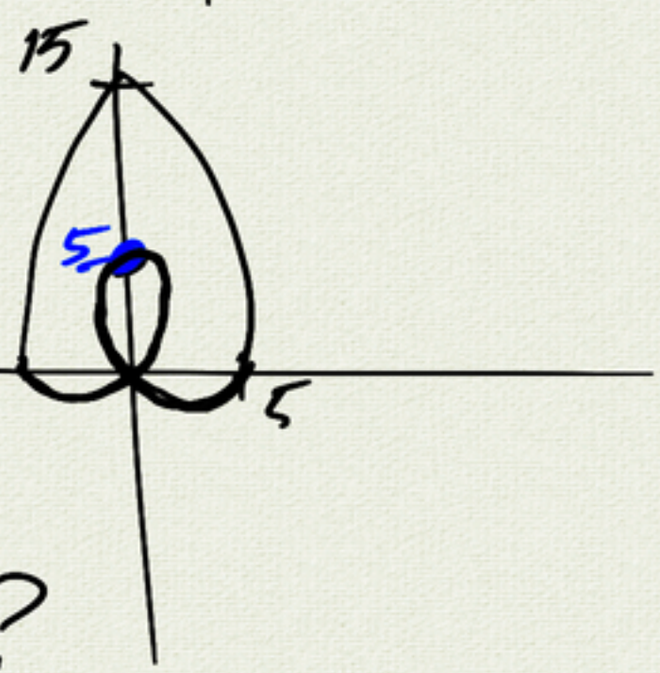
θ	$5\sin\theta$	$5+5\sin\theta$
0	0	5
$\pi/2$	5	10
π	0	5
$3\pi/2$	-5	0
2π	0	5



cardioid

$r = 5 + 10\sin\theta$

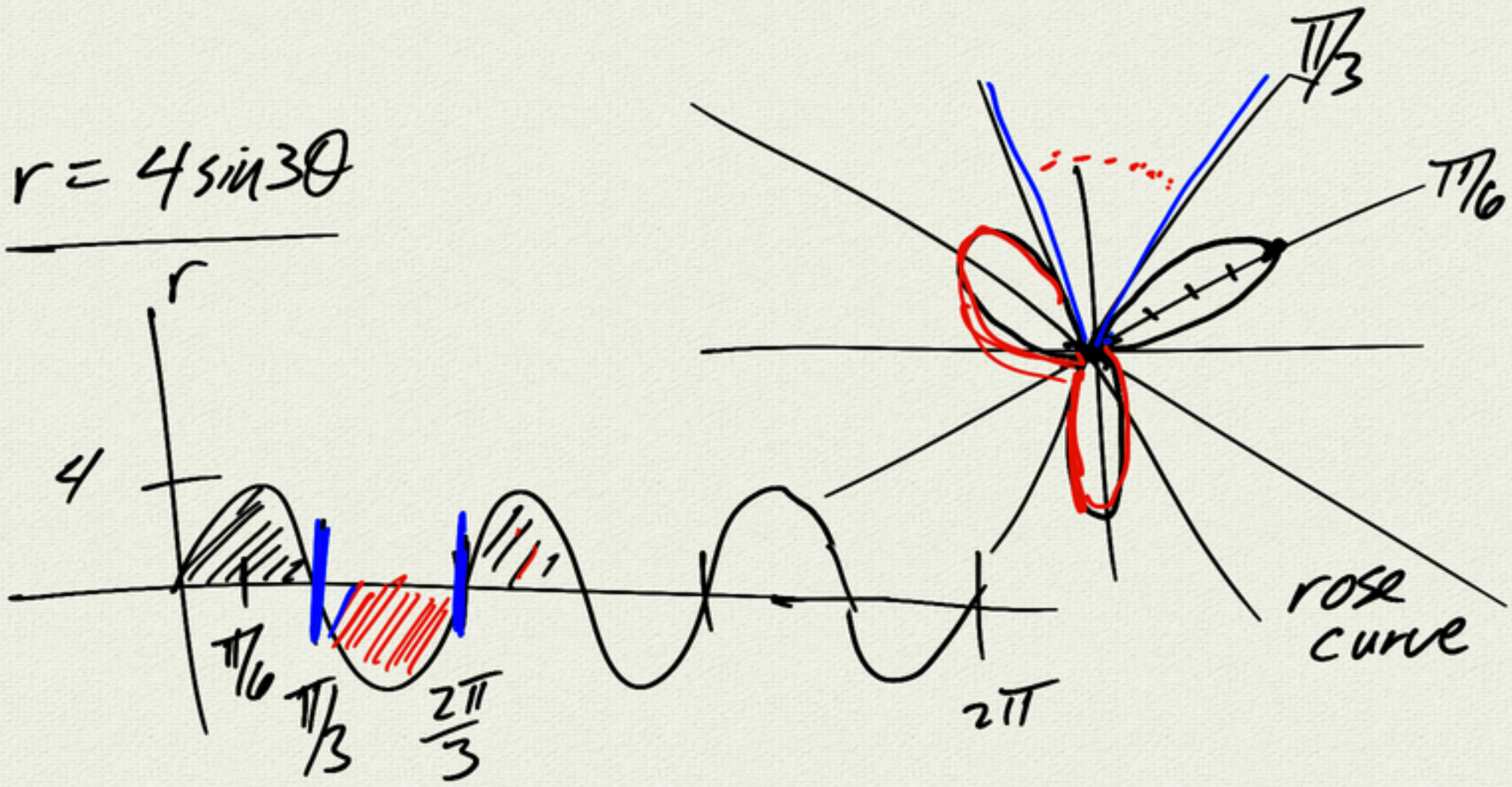
θ	$10\sin\theta$	r
0	0	5
$\pi/2$	10	15
π	0	5
$3\pi/2$	-10	-5
2π	0	5



challenge: where is $r=0$?

limaçon

$r = 4\sin 3\theta$

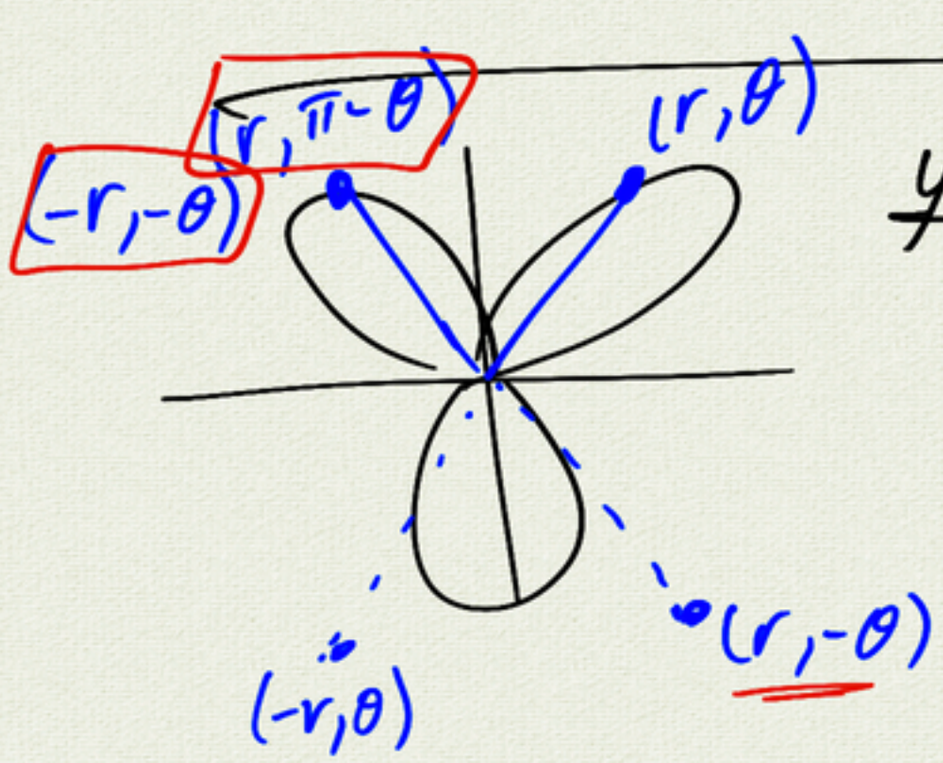


① graph on Desmos or calc

② max |r| value

$r = 4\sin 3\theta$
$\Rightarrow \max|r| = 4$

③ symmetry



y-axis symmetry

equation $r = 4\sin 3\theta$

check symmetric pt:

$(r, \pi-\theta)$:

$r \stackrel{?}{=} 4\sin[3(\pi-\theta)]$

$\stackrel{?}{=} 4\sin(3\pi - 3\theta)$

$\stackrel{?}{=} 4\sin(\pi - 3\theta)$?

$(-r, -\theta)$

$-r \stackrel{?}{=} 4\sin(3(-\theta))$

$= -4\sin 3\theta$ ✓

(sin is odd

symmetry

x-axis

y-axis

origin (180° rotation)