3.6 Linear Systems

$$3x + 9y = 21$$
$$-2x - 5y = -12$$

$\Downarrow$ augmented matrix

$$\left[\begin{array}{cc|c} 3 & 9 & 21 \\ -2 & -5 & -12 \end{array}\right]$$

goal $\Longrightarrow$ $\left(\begin{array}{cc|c} 1 & 0 & a \\ 0 & 1 & b \end{array}\right)$ $x = a$, $y = b$

$\frac{1}{3}R_1$ $\Downarrow$

$$\left(\begin{array}{cc|c} 1 & 3 & 7 \\ -2 & -5 & -12 \end{array}\right)$$

$2R_1 + R_2$ $\Downarrow$

$$\left(\begin{array}{cc|c} 1 & 3 & 7 \\ 0 & 1 & 2 \end{array}\right)$$

$\Downarrow$

$-3R_2 + R_1$ $\left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 2 \end{array}\right)$ $\Longrightarrow$ $x = 1$, $y = 2$

example 2

$$3x + y + 3z = 14$$
$$x + 2z = 7$$
$$-x + 2y - 7z = -18$$

$$\left(\begin{array}{ccc|c} 3 & 1 & 3 & 14 \\ 1 & 0 & 2 & 7 \\ -1 & 2 & -7 & -18 \end{array}\right) \Rightarrow \text{goal} \left(\begin{array}{ccc|c} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \end{array}\right)$$

R_{12}

$$\left(\begin{array}{ccc|c} \boxed{1} & 0 & 2 & 7 \\ 3 & 1 & 3 & 14 \\ -1 & 2 & -7 & -18 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & \boxed{1} & -3 & -7 \\ 0 & 2 & -5 & -11 \end{array}\right)$$

$-3R_1 + R_2$
$1 \cdot R_1 + R_3$

$$\left(\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & -3 & -7 \\ 0 & 0 & 1 & 3 \end{array}\right)$$

$-2R_2 + R_3$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right) \Rightarrow \begin{array}{l} x = 1 \\ y = 2 \\ z = 3 \end{array}$$

$-2R_3 + R_1$
$3R_3 + R_2$

check answer by plugging in!