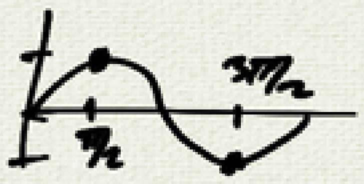
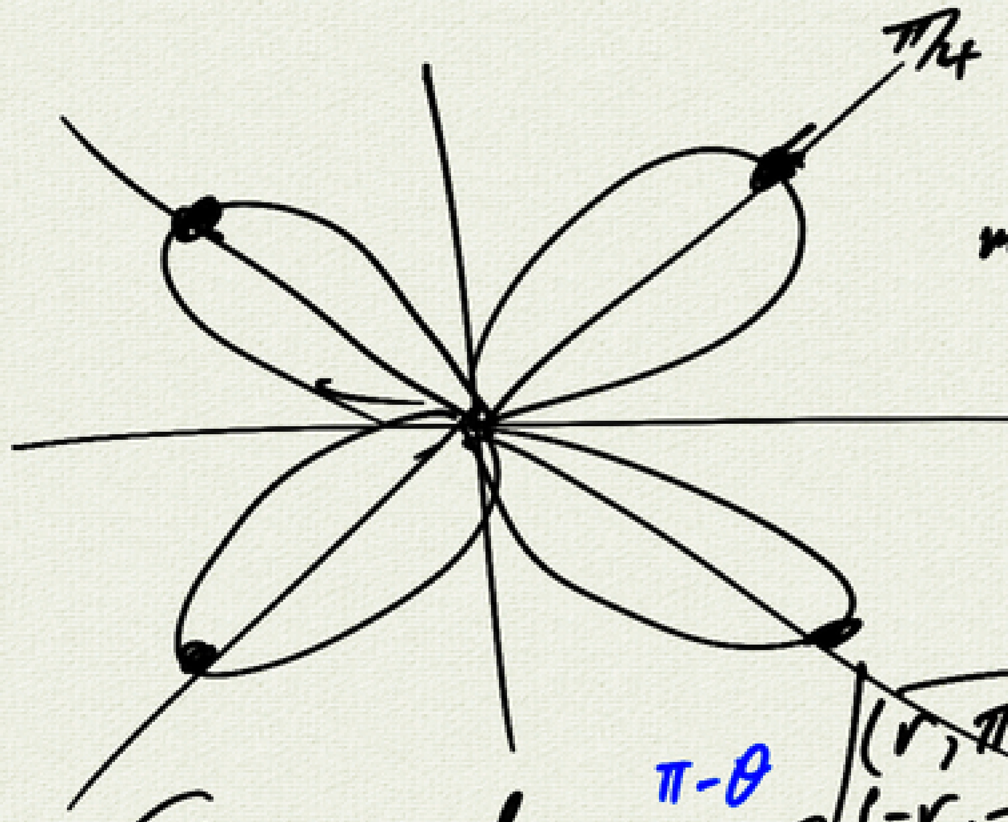
(9) $r = 3\sin 2\theta$ ← (1) max|r| value 



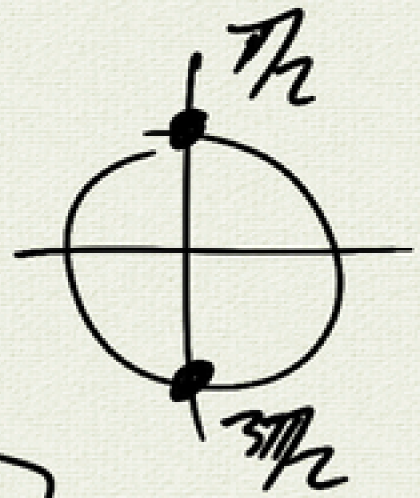
$|r| = |3\underbrace{\sin 2\theta}_{\text{in } [-1,1]}|$

max $|r| = 3$ when

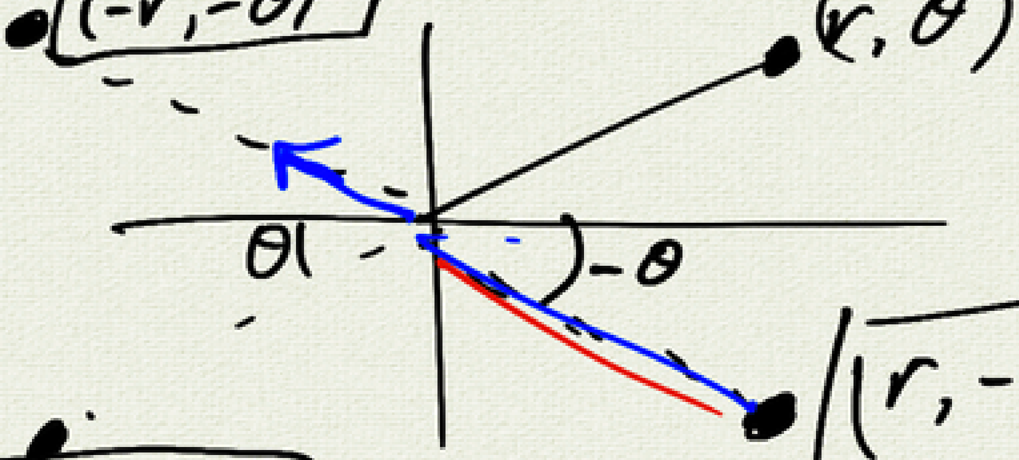
$\sin 2\theta = \pm 1$

$\to 2\theta = \frac{\pi}{2} + k\pi$

$\boxed{\theta = \frac{\pi}{4} + k\frac{\pi}{2}}$



(2) symmetry



$\boxed{\begin{matrix}(r, \pi-\theta) \\ (-r, -\theta)\end{matrix}}$ y-axis

$\boxed{\begin{matrix}(-r, \theta) \\ (r, \pi+\theta)\end{matrix}}$ origin

$\boxed{\begin{matrix}(r, -\theta) \\ (-r, \pi-\theta)\end{matrix}}$ x-axis

x-axis

$r = 3\sin 2\theta$

$(r, -\theta)$: $r \neq 3\sin 2\theta$ X

$r \overset{?}{=} 3\sin 2(-\theta)$

$(-r, \pi-\theta)$: $-r \overset{?}{=} 3\sin[2(\pi-\theta)]$

$-r \overset{?}{=} 3\sin(2\pi - 2\theta)$

$-r = 3\sin(-2\theta)$

$-r \overset{?}{=} -3\sin 2\theta$

$r = 3\sin 2\theta$ ✓

$\boxed{\begin{matrix}\sin(x+2\pi) = \sin x \\ \sin(x+100\pi) = \sin x\end{matrix}}$

origin try $(-r, \theta)$, $(r, \pi+\theta)$

$r = 3\sin 2\theta$ $(-r, \theta)$: $-r \overset{?}{=} 3\sin 2\theta$ X

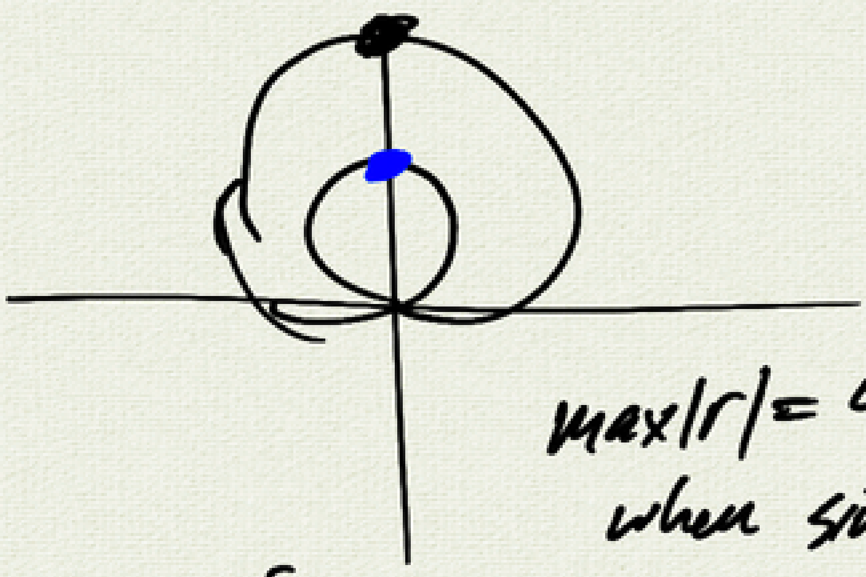
$(r, \pi+\theta)$: $r \overset{?}{=} 3\sin[2(\pi+\theta)]$

$= 3\sin(2\pi + 2\theta)$

$= 3\sin 2\theta$ ✓

---

(27) $r = 1 + 3\sin\theta$



(1) max|r|

$|r| = |1 + \underbrace{3\sin\theta}_{\text{in } [-1,1]}|$

max: $1+3 = 4$ ← $\sin\theta = 1$

min: $|1-3| = 2$ ← $\sin\theta = -1$, $\theta = \pi/2$

max $|r| = 4$

when $\sin\theta = 1$

$\theta = \pi/2$

3.7 Matrix algebra

matrix $\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ $\begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

$\begin{pmatrix} 1 & 3 & 5 \\ 2 & 4 & 6 \end{pmatrix}$

$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{pmatrix}$ a_{ij}

row

column

row 2

column 3

2×3 matrix (2 rows, 3 columns)

---

2 operations $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$ $B = \begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix}$

addition: $A + B = \begin{pmatrix} 4 & 8 \\ 6 & 10 \end{pmatrix}$

A, B must be same size

scalar multiplication $2A = 2\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$

$= \begin{pmatrix} 2 & 6 \\ 4 & 8 \end{pmatrix}$

---

matrix multiplication

$1\cdot 3 + 3\cdot 4$

$1\cdot 5 + 3\cdot 6$

$$\begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 3 & 5 \\ 4 & 6 \end{pmatrix} = \begin{pmatrix} \boxed{15} & \boxed{23} \\ \boxed{22} & 34 \end{pmatrix}$$

A B

$$\begin{pmatrix} 1 & 3 & 1 \\ 2 & 4 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \end{pmatrix} = \begin{pmatrix} \square & \\ & \end{pmatrix}$$

2×3 3×2 2×2

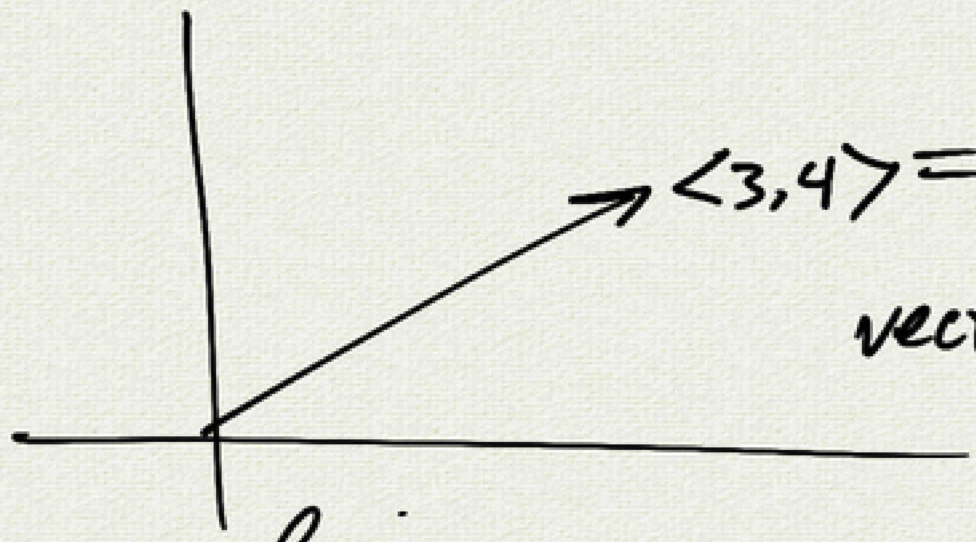
$(4\times 3)\cdot(3\times 2) \Longrightarrow 4\times 2$

$$\begin{pmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{pmatrix}\begin{pmatrix} \bullet & \bullet \\ \bullet & \bullet \\ \bullet & \bullet \end{pmatrix} = \begin{pmatrix} \bullet & \bullet \\ \bullet & \bullet \\ \bullet & \bullet \\ \bullet & \bullet \end{pmatrix}$$

$$\begin{pmatrix} \bullet & \bullet \\ \bullet & \bullet \end{pmatrix}\begin{pmatrix} \bullet & \bullet \\ \bullet & \bullet \end{pmatrix} = \begin{pmatrix} \bullet & \bullet \\ \bullet & \bullet \end{pmatrix}$$

$\langle 3,4\rangle = \begin{pmatrix}3\\4\end{pmatrix}$ ← column vector

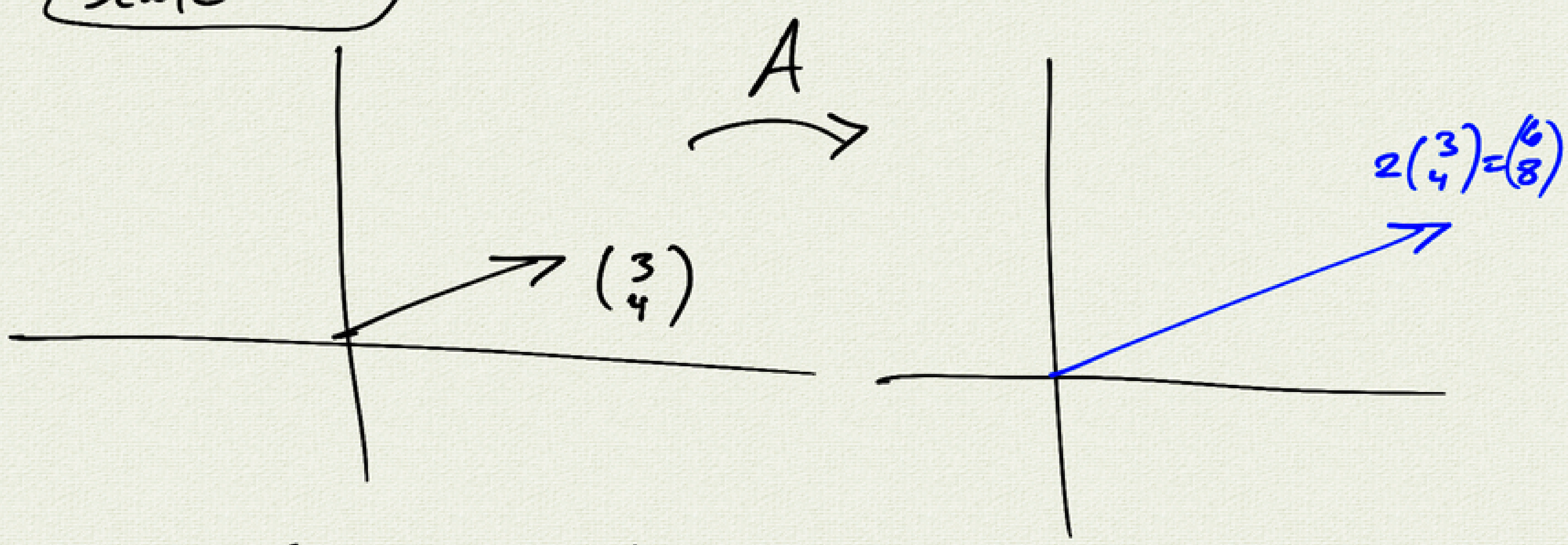
vector notation



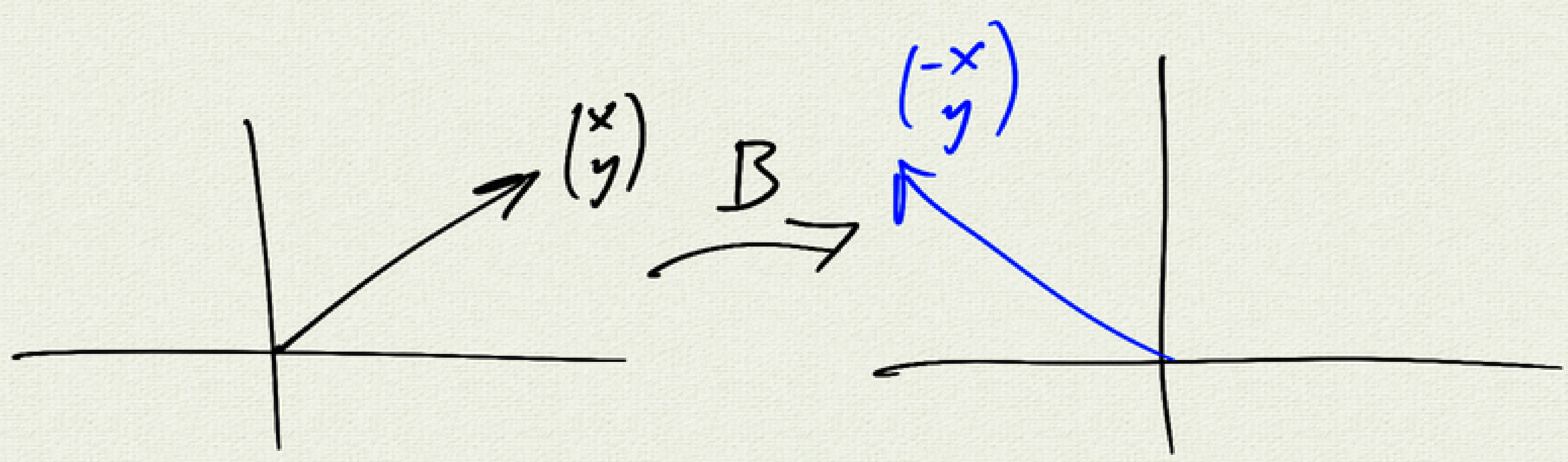
linear transformation

examples:

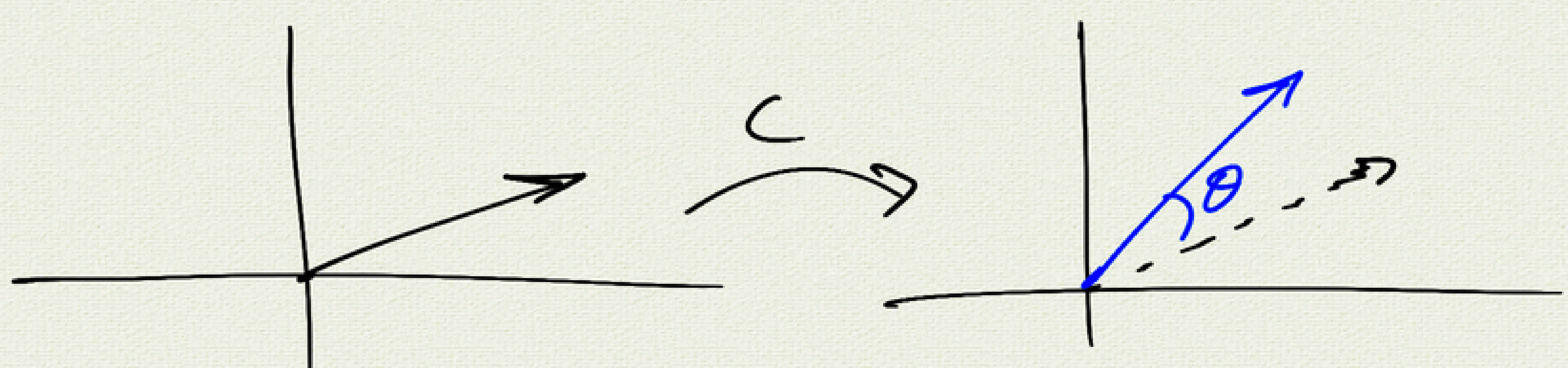
Scale ×2



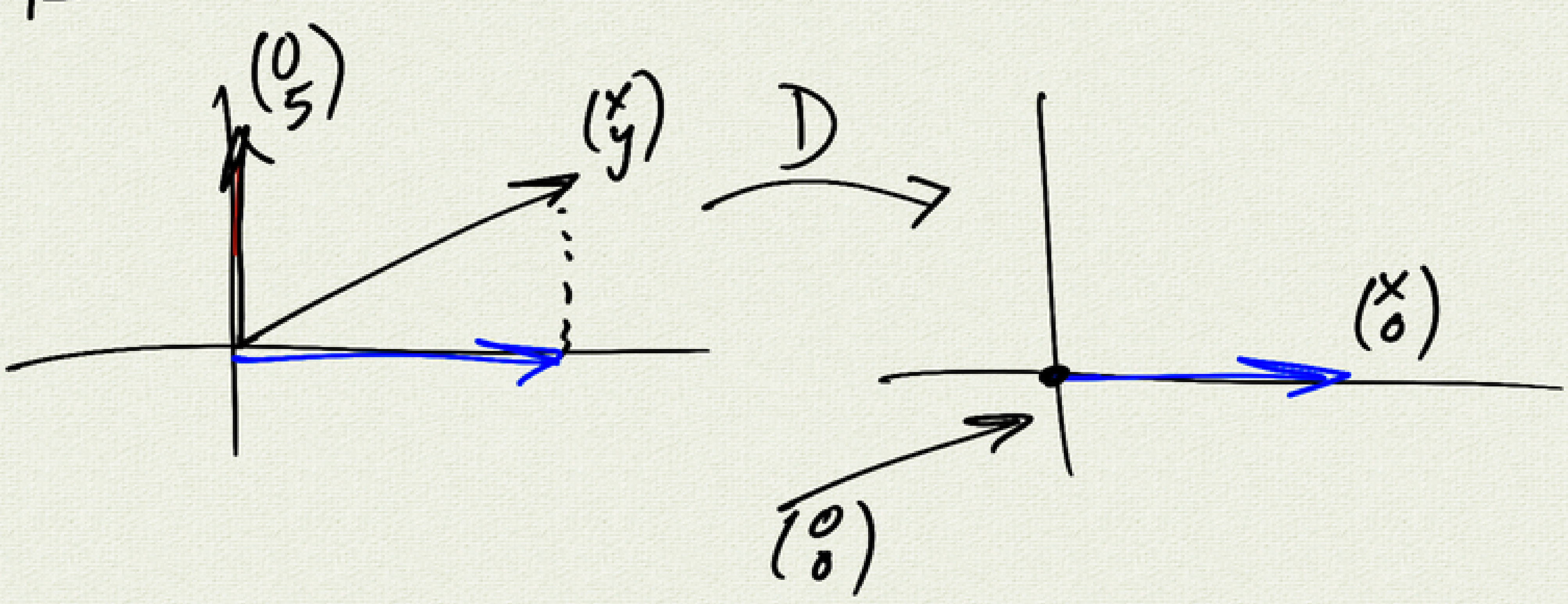
reflection across y-axis

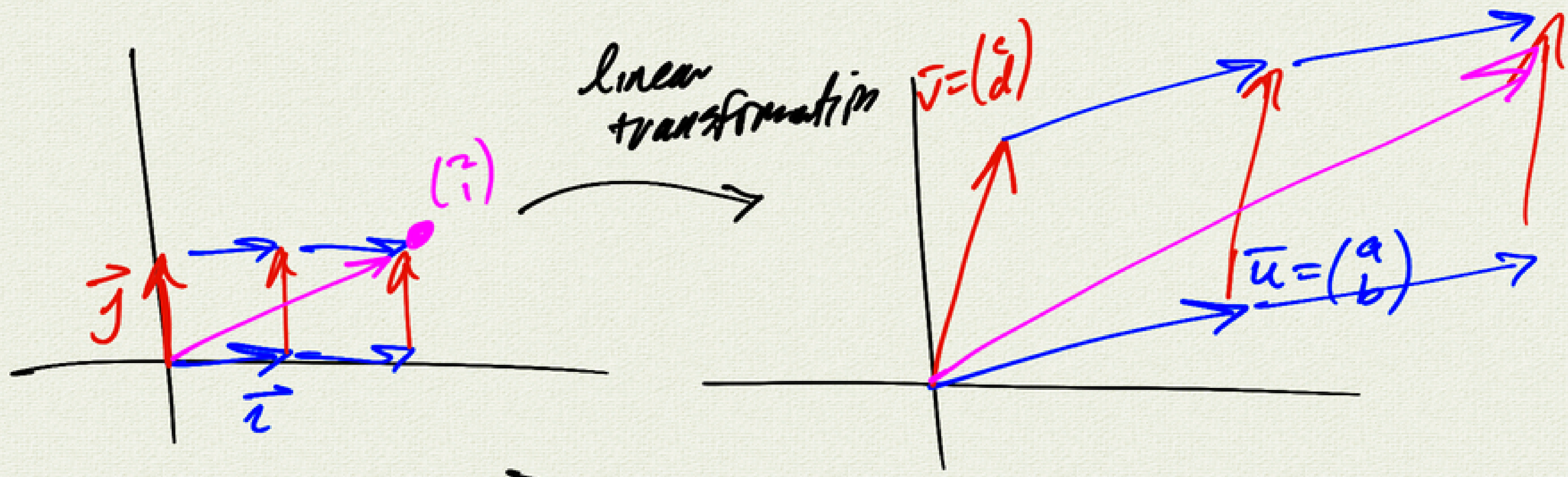


rotation by θ



projection onto x-axis





$$\vec{i} \longrightarrow \vec{u}$$
$$\vec{j} \longrightarrow \vec{v}$$

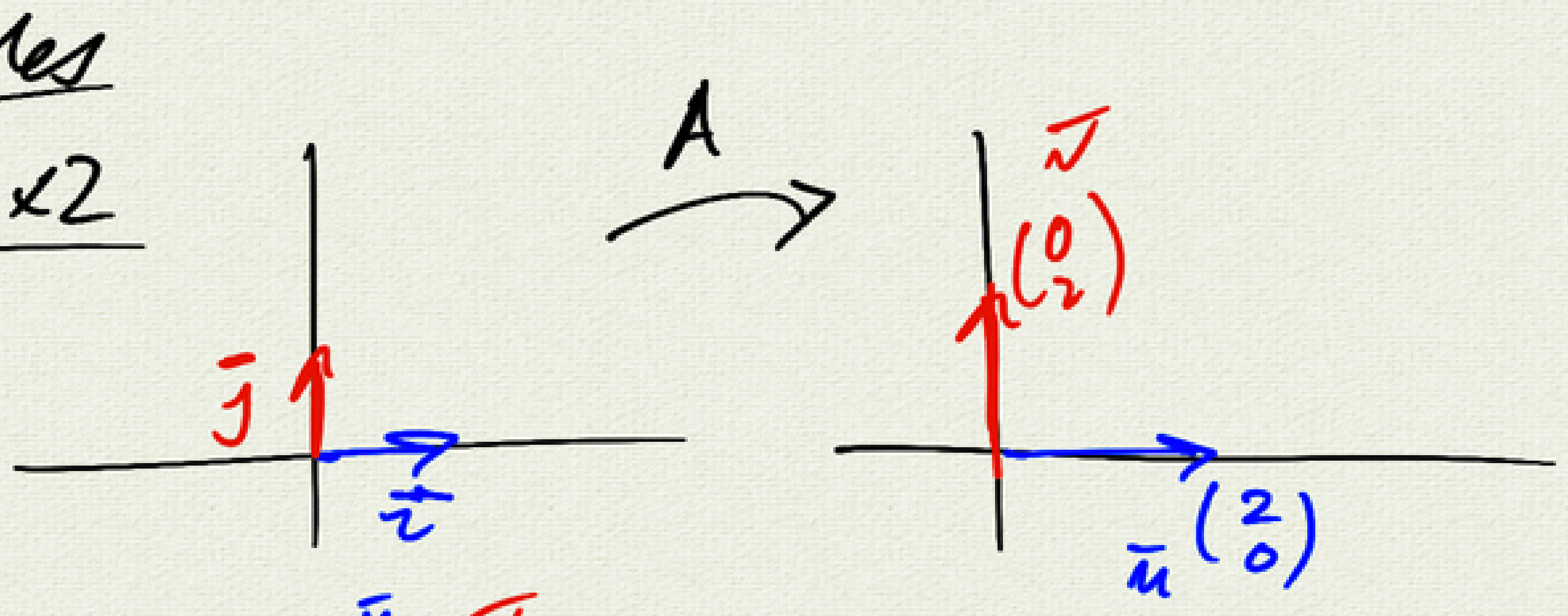
$$\begin{pmatrix}2\\1\end{pmatrix} = 2\vec{i} + 2\vec{j} \longrightarrow 2\vec{u} + 2\vec{v}$$

$$\begin{pmatrix}x\\y\end{pmatrix} = x\vec{i} + y\vec{j} \longrightarrow x\vec{u} + y\vec{v}$$
$$= x\begin{pmatrix}a\\b\end{pmatrix} + y\begin{pmatrix}c\\d\end{pmatrix}$$
$$= \begin{pmatrix}xa + yc\\ xb + yd\end{pmatrix}$$
$$= \begin{array}{cc} \vec{u} & \vec{v} \\ \end{array}$$
$$\begin{pmatrix}a & c\\ b & d\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$$

linear transformation is specified by matrix

applying the transformation = matrix multiplication examples

Scale ×2

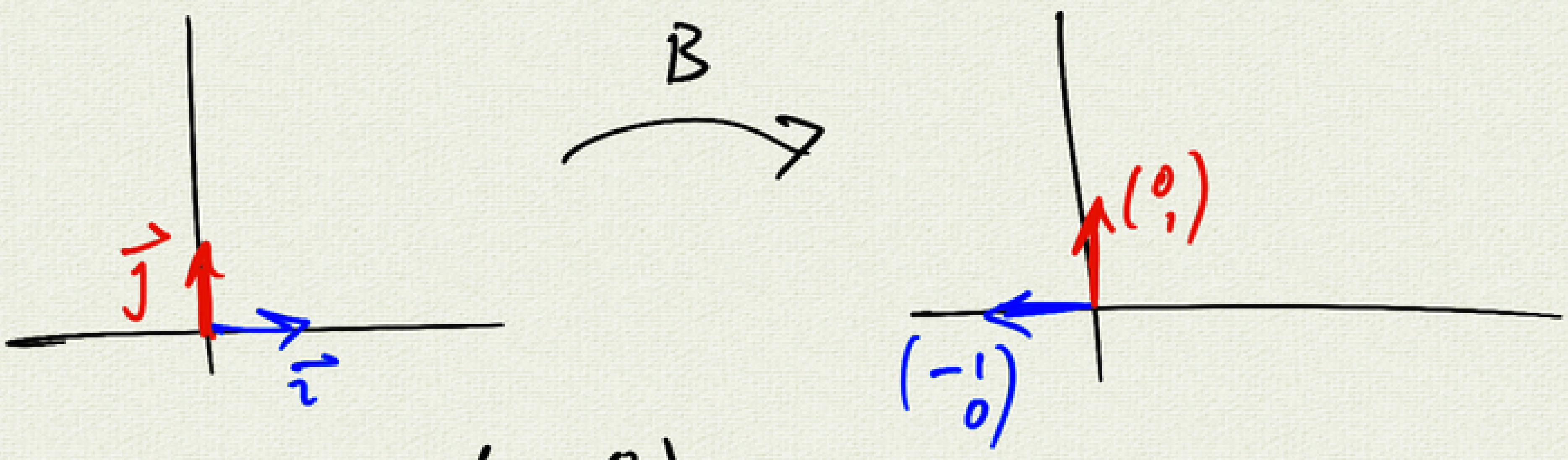


$$A = \begin{array}{c} \begin{matrix} \vec u & \vec v \end{matrix} \\ \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} \end{array}$$

then $A\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2x \\ 2y \end{pmatrix}$

2× original vector

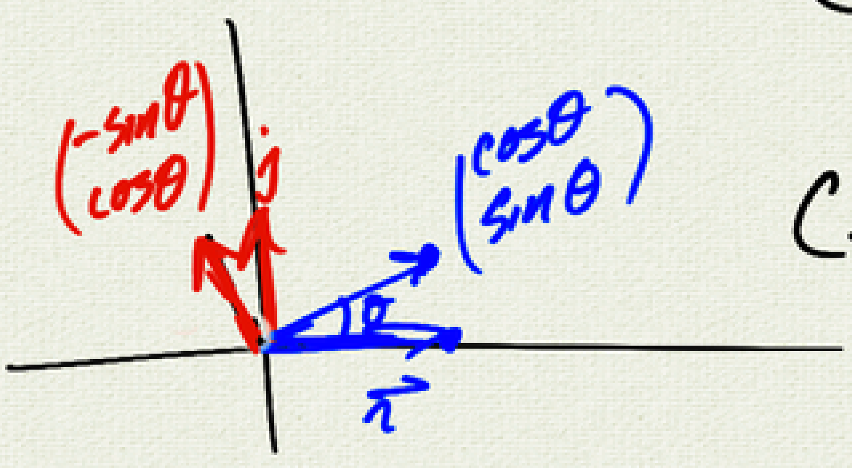
reflection across y-axis



$$B = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$B\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -x \\ y \end{pmatrix}$$

rotation by θ



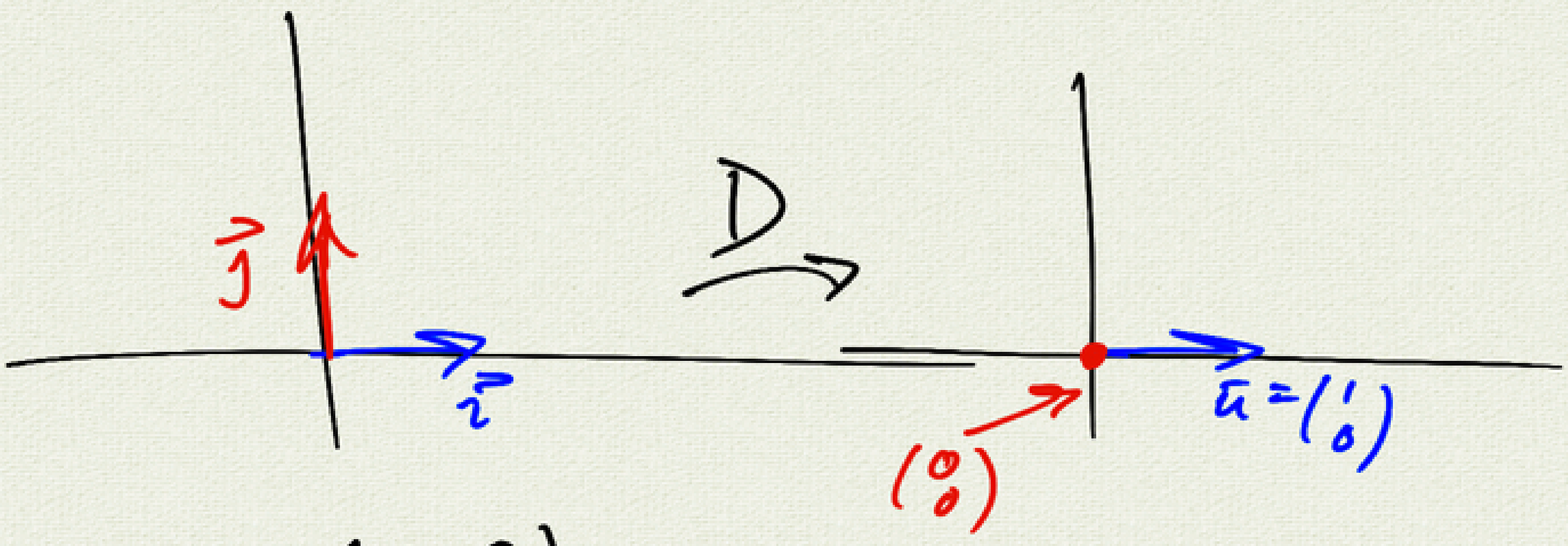
$$C_\theta = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$C_{\pi/2} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$C_{\pi/2}\,\vec\imath \stackrel{?}{=} \vec\jmath$$

Check: $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ ✓

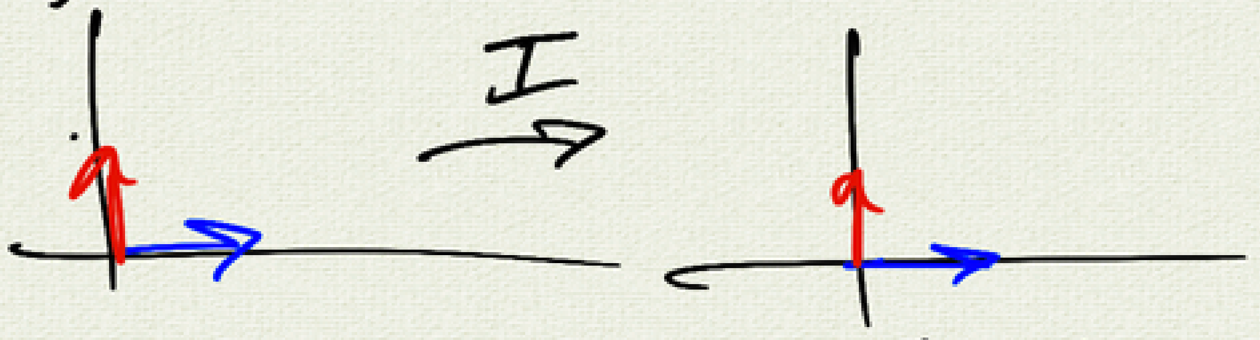
projection on x-axis



$$D = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

check $D\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ 0 \end{pmatrix}$

identity transformation



$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ identity matrix

$$I\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

also: $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$

$$AI = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} = A$$

$$IA = A$$