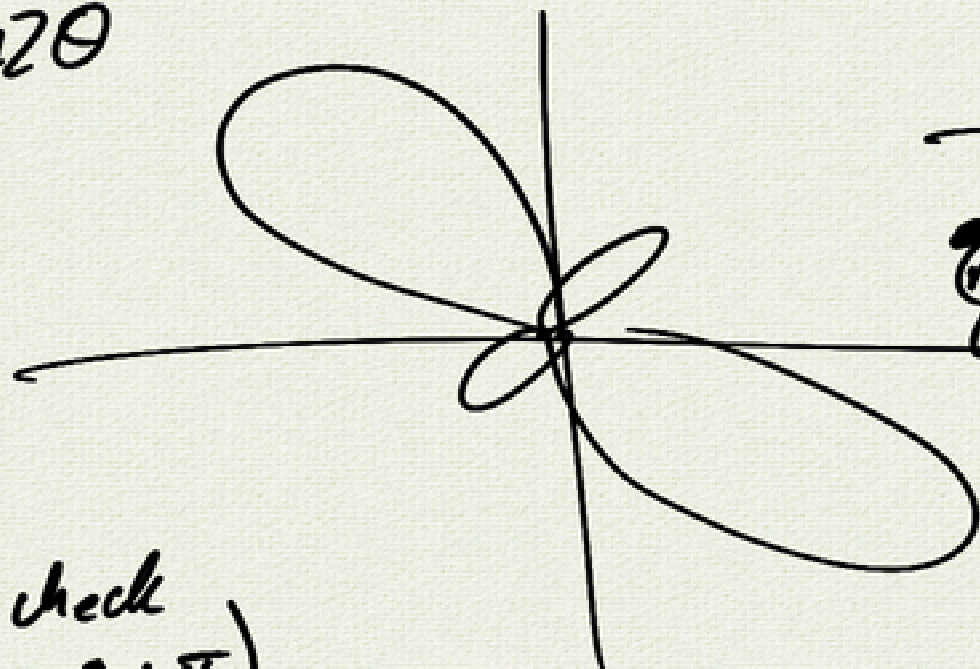
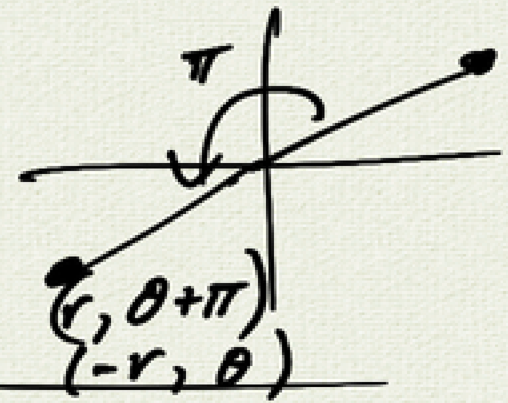
$r = 1 - 2\sin 2\theta$



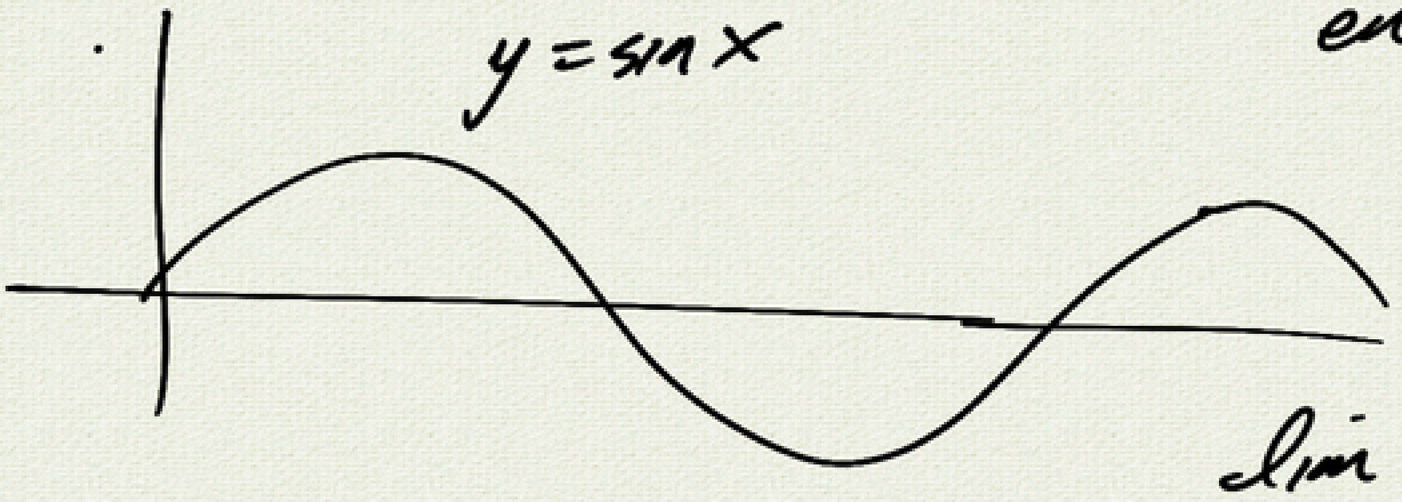


<u>origin</u>: check $(r, \theta + \pi)$

$$r \stackrel{?}{=} 1 - 2\sin(2(\theta+\pi))$$
$$= 1 - 2\sin(2\theta + 2\pi)$$
$$= 1 - 2\sin 2\theta \checkmark$$

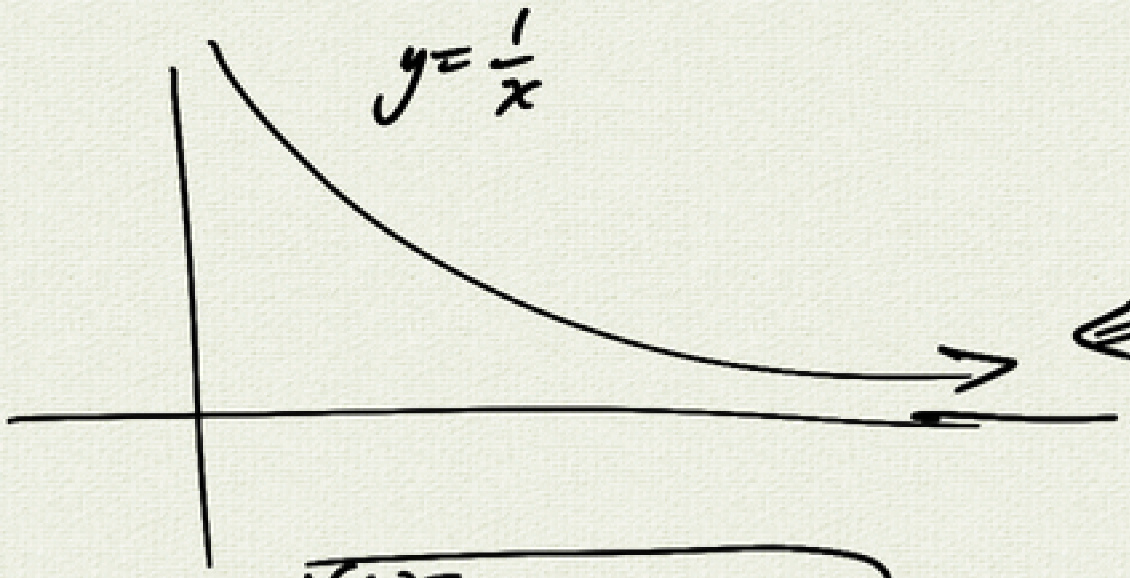
$y = \sin x$

end behavior?



$\lim\limits_{x\to\infty} \sin x$ does not exist

$y = \frac{1}{x}$

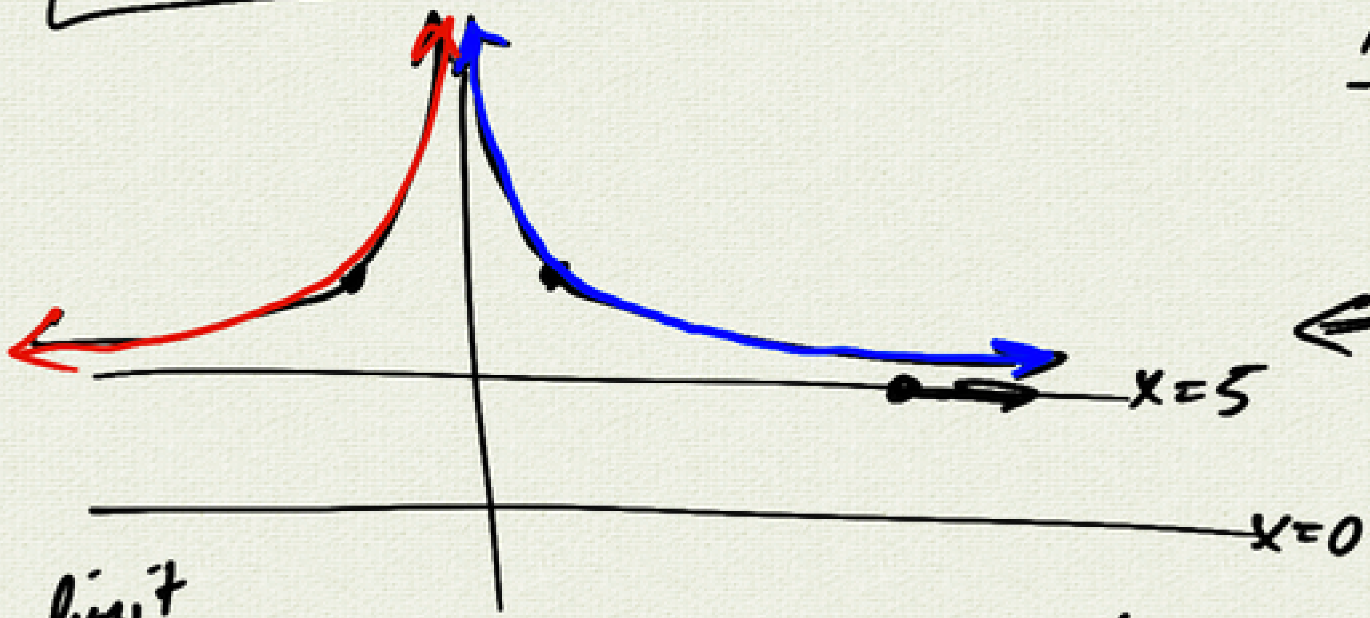


$\Leftarrow \lim\limits_{x\to\infty} \frac{1}{x} = 0$

x	$1/x^2$
1	1
10	$1/10^2$
100	$1/100^2$

x	$1/x^2 + 5$
.1	$100 + 5$
.01	$100^2 + 5$
.001	$1000^2 + 5$

$\boxed{f(x) = \; y = \frac{1}{x^2} + 5}$



end behavior $\Leftarrow$

$\lim\limits_{x\to\infty} f(x) = 5$

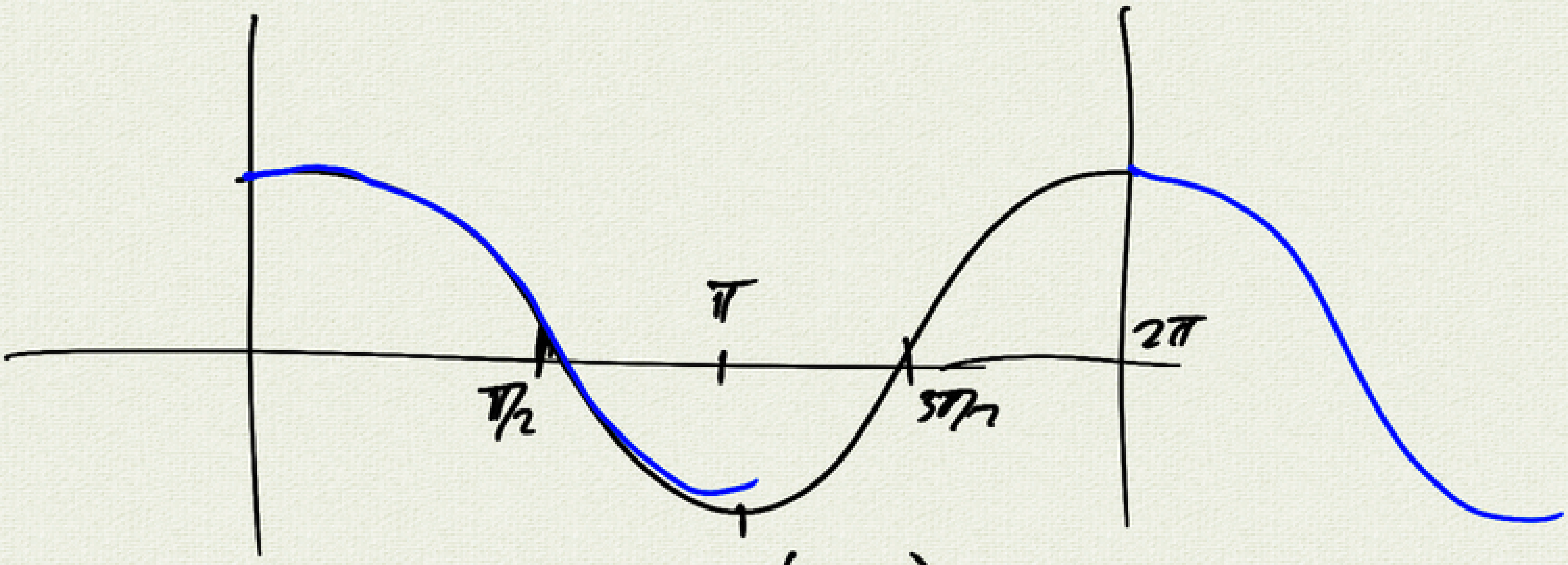
limit

$\Rightarrow \lim\limits_{x\to 0} f(x) = \infty$

<u>decreasing</u>: $(0, \infty)$

<u>increasing</u>: $(-\infty, 0)$

$g(x) = \cos x$



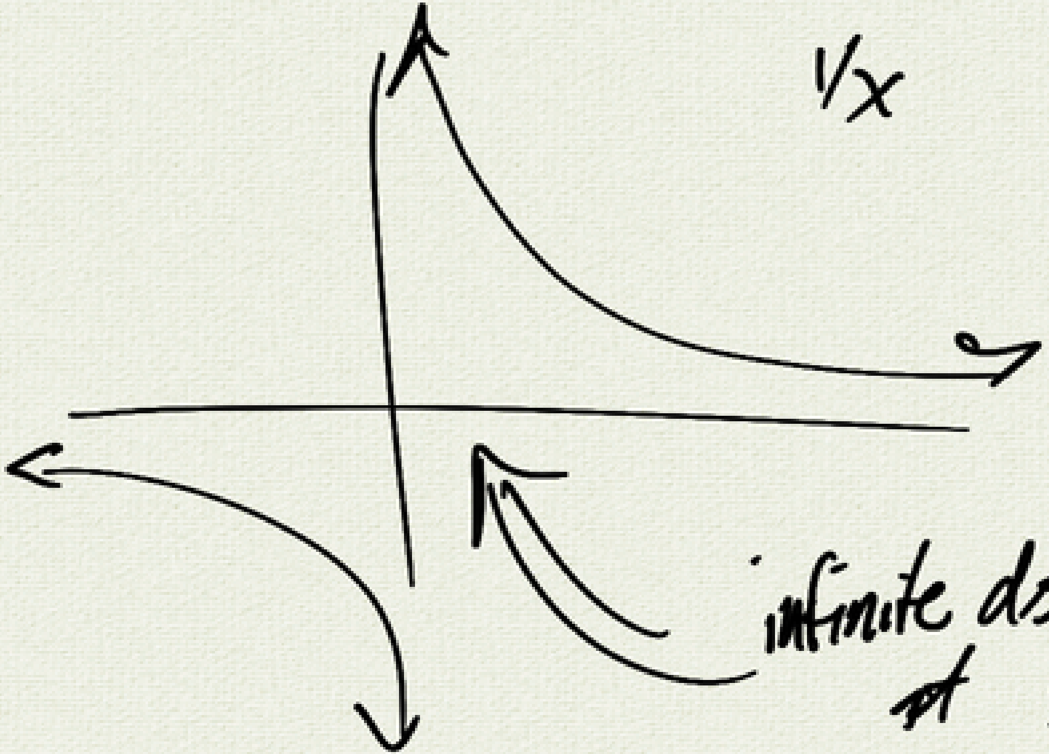
decreasing on $[0, \pi]$ $(+k2\pi)$

$\longrightarrow$ $[2k\pi, 2k\pi + \pi]$ for $k \in \mathbb{Z}$

$\Rightarrow$ <u>$k=0$</u> $[0, \pi]$

<u>$k=1$</u> $[2\pi, 3\pi]$

$1/x$



infinite discontinuity
at $x = 0$

4.2 Function operations

f, g functions $\quad f: \mathbb{R} \to \mathbb{R}$
$\quad g: \mathbb{R} \to \mathbb{R}$

$\Rightarrow (f+g)(x) = f(x) + g(x)$

example:
$f(x) = \sin x$
$g(x) = 5$
$(f+g)(x) = (\sin x) + 5$

$\Rightarrow (fg)(x) = f(x)g(x)$

$\Rightarrow (f/g)(x) = f(x)/g(x)$

example: $f(x) = 5$
$g(x) = x^2$
$(f/g)(x) = \frac{5}{x^2} \Leftarrow$ domain $g(x) \neq 0$
$x^2 \neq 0$
$x \neq 0$

$(f \circ g)(x) = f(g(x))$
composition

example: $f(x) = \sin x$
$g(x) = 2x$
$(f \circ g)(x) = f(g(x))$
$= \sin(2x)$
$(g \circ f)(x) = g(f(x))$
$= 2(\sin x)$

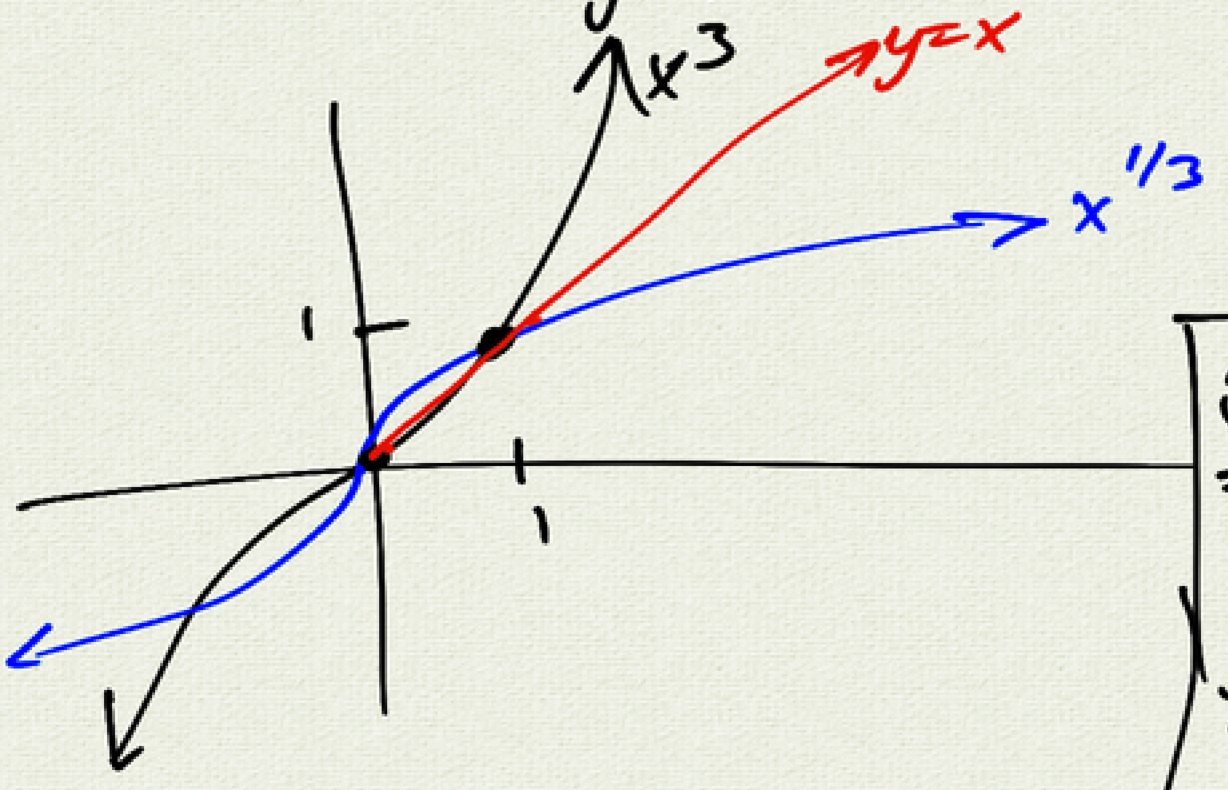
---

inverse functions: f, g are inverse functions
if $(f \circ g)(x) = x$
and $(g \circ f)(x) = x$

---

examples: $f(x) = x^3$
$g(x) = x^{1/3}$

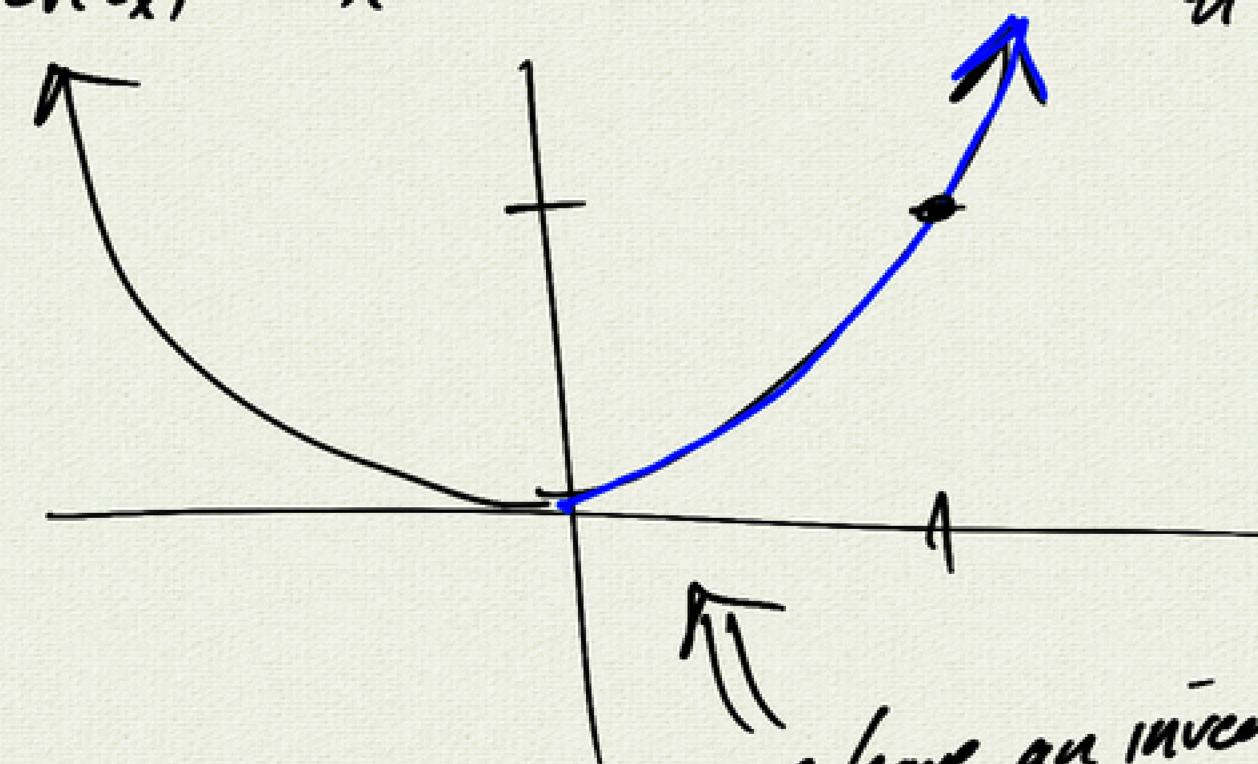
x	x^3
1	1
2	8
3	27



given $f(x) = x^3$
$\Rightarrow$ find $f^{-1}(x)$?
$y = x^3$
swap x, y
$x = y^3$
solve for y:
$y = x^{1/3}$
$f^{-1}(x) = x^{1/3}$

$h(x) = x^2$

$h^{-1}(x)$?

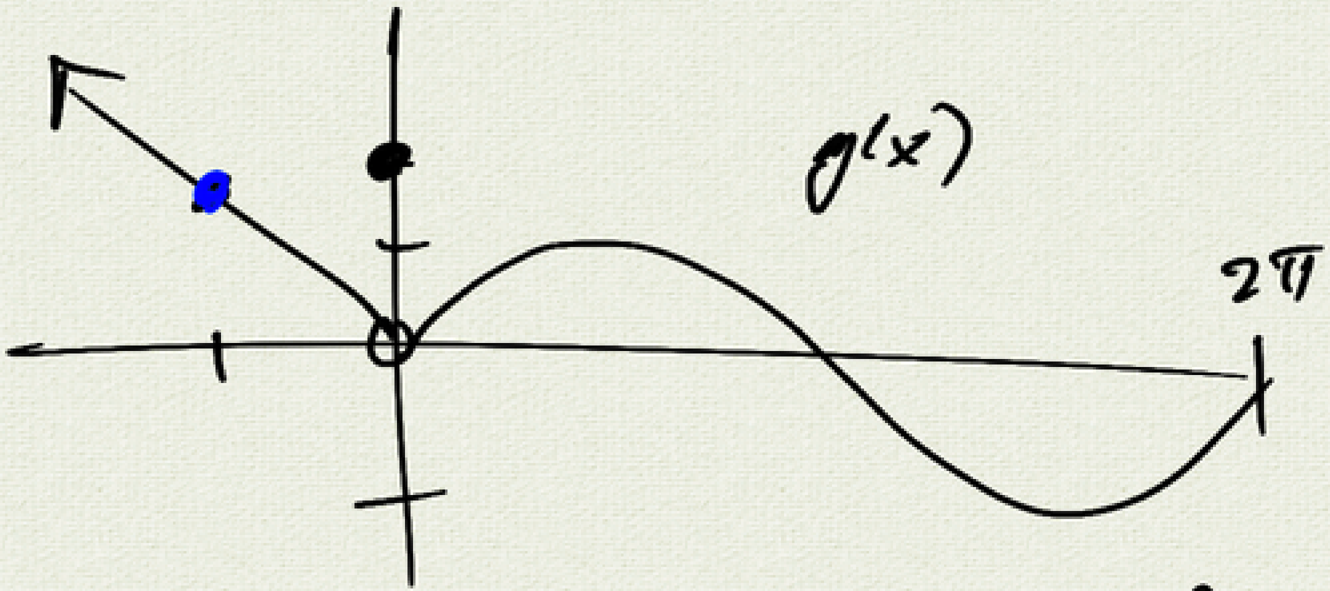


x	$h(x)=x^2$
0	0
± 1	1
± 2	4
± 3	9

we have an inverse
if we restrict domain
so that the function is 1-1
(horizontal line test)

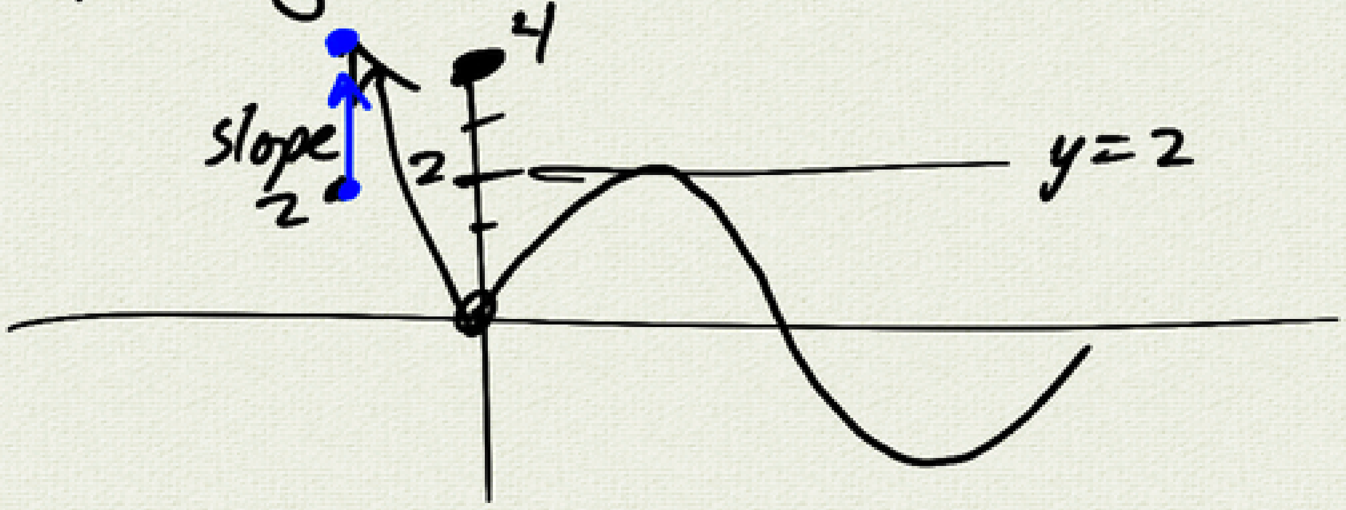
(for a y-value, there is a unique x-value)

$$g(x) = \begin{cases} -x & x<0 \\ 2 & x=0 \\ \sin x & x>0 \end{cases}$$



$h(x) = g(x)+3$ (vertical shift 3)

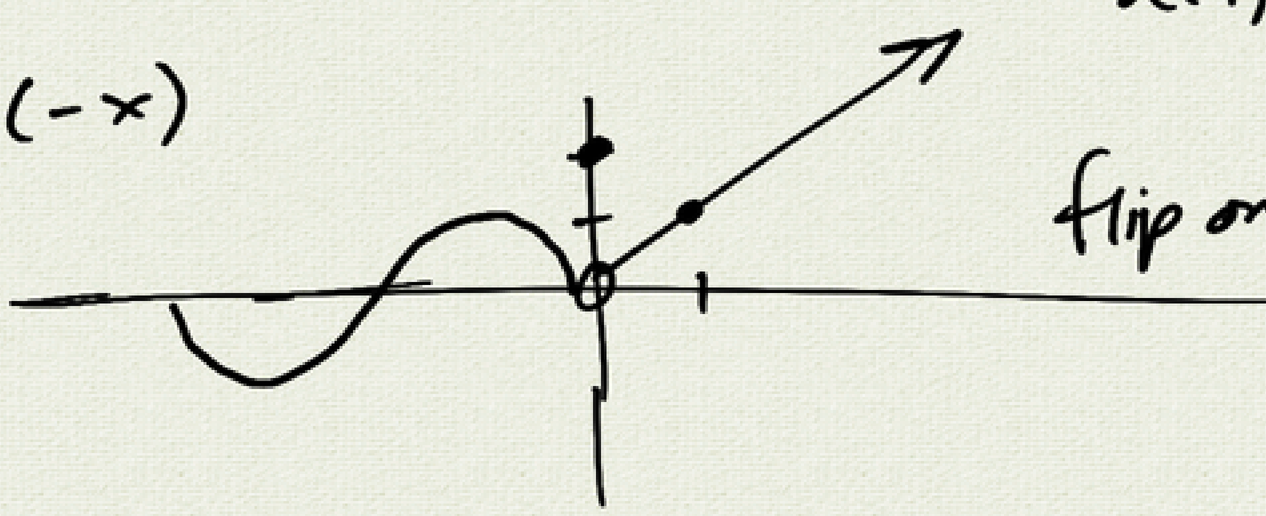
$k(x) = 2g(x)$ (vertical scale x2)



$l(1) = g(-1)$

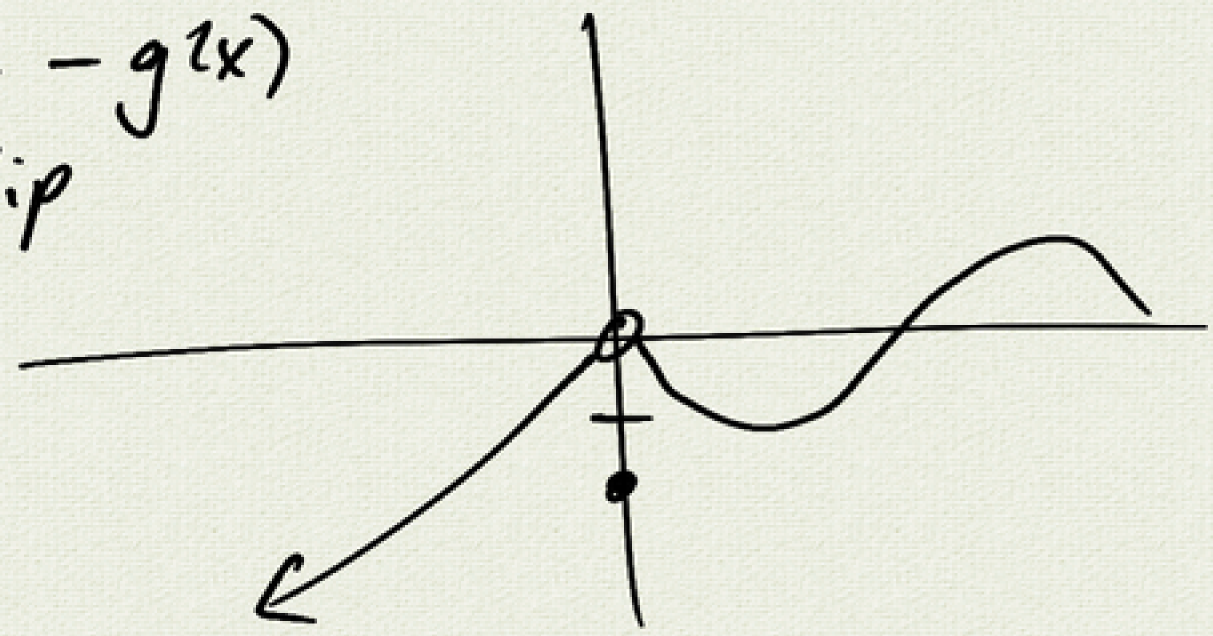
$l(x) = g(-x)$

flip over y-axis

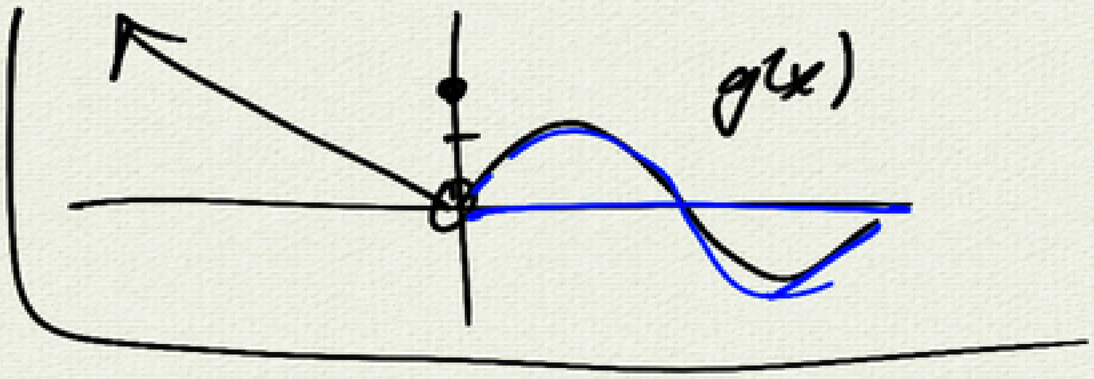


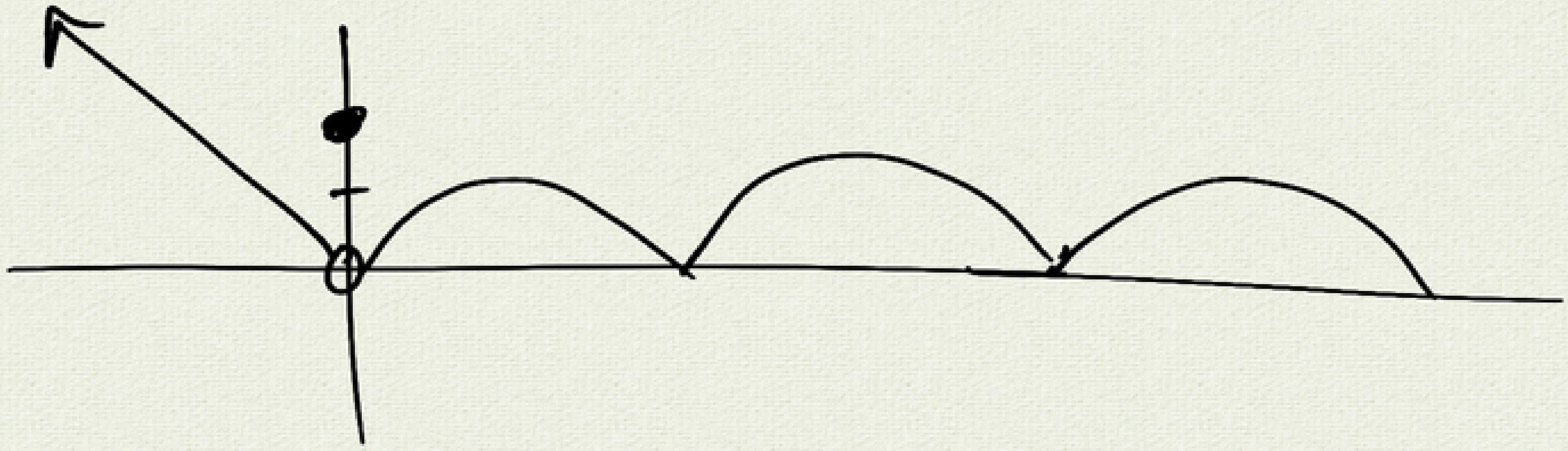
$m(x) = -g(x)$

vertical flip



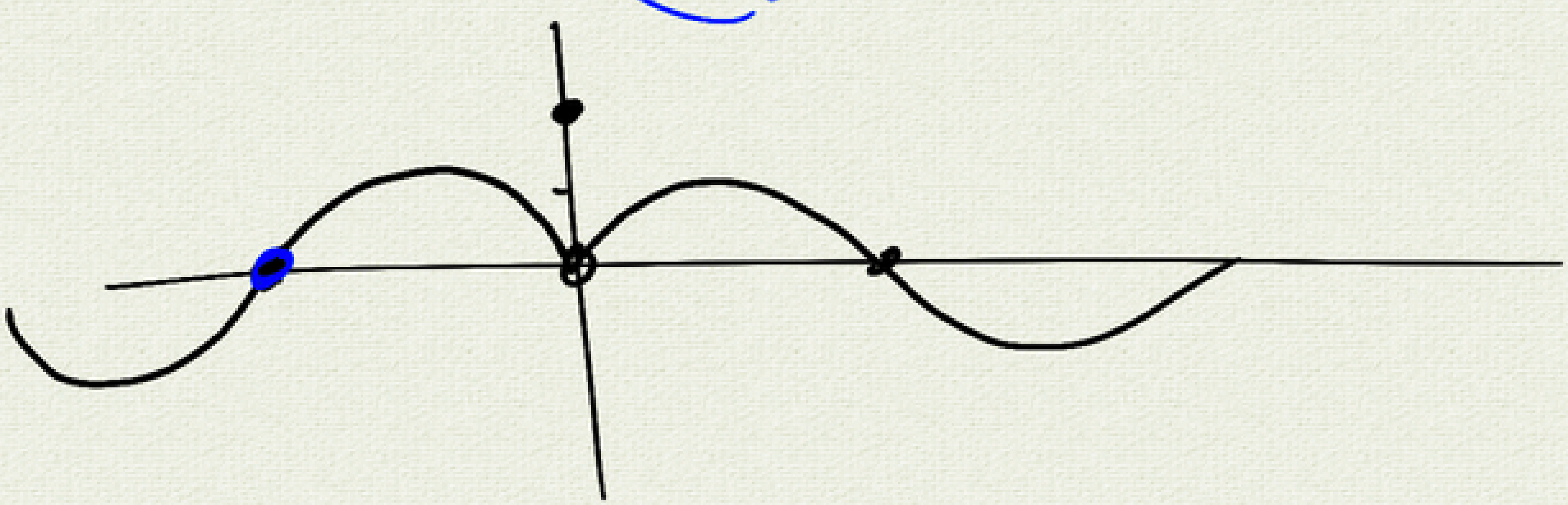
$h(x) = |g(x)|$





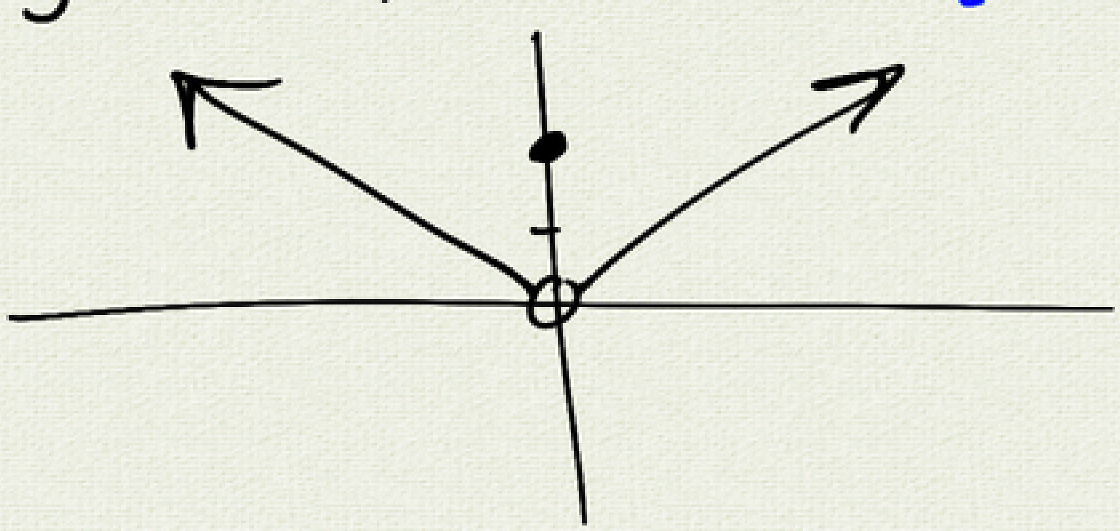
$p(x) = g(|x|)$ always > 0

$p(-\pi) = g(|-\pi|)$
$= g(\pi)$

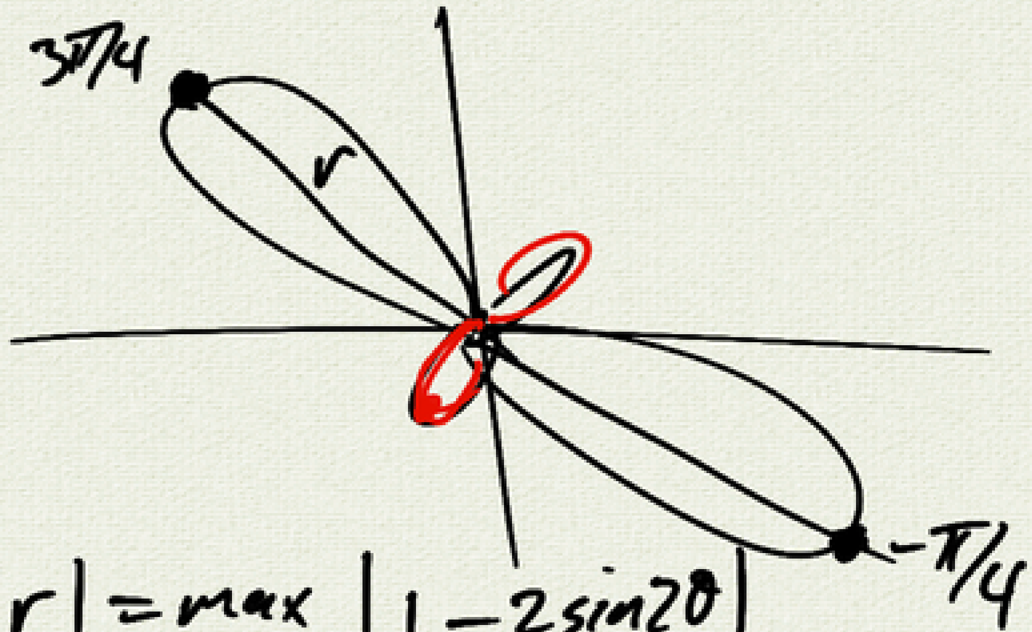


x	$p(x)$
$-\pi$	0

$q(x) = g(-|x|)$ $\Leftarrow$ for $x<0$ $q(x) = g(x)$



$r = 1 - 2\sin 2\theta$

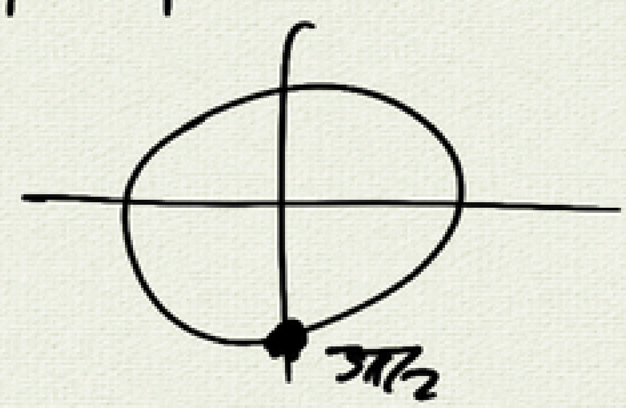


θ	r
0	1
	max

$$\max|r| = \max\left|1 - 2\underbrace{\sin 2\theta}_{[-1,1]}\right|$$

$|1 - 2(-1)|$ vs. $|1 - 2(1)|$ challenge

$|3|$ biggest $\qquad |-1|$



$\boxed{\text{when } \sin 2\theta = -1}$

$$2\theta = \frac{3\pi}{2} + 2\pi k$$

$$\theta = \frac{3\pi}{4} + \pi k$$

$1 - 2\sin 2\theta$