multiplicity

$\boxed{3.4}$

57

deg 3

zeros: $-2, 1, 3$

y-intercept -4

Factor Theorem:

a is a zero $\Longleftrightarrow$ $x-a$ is a factor

$f(a)=0 \Longleftrightarrow x-a \mid f(x)$

"divides"

factors:

$$(x+2)(x-1)(x-3)$$

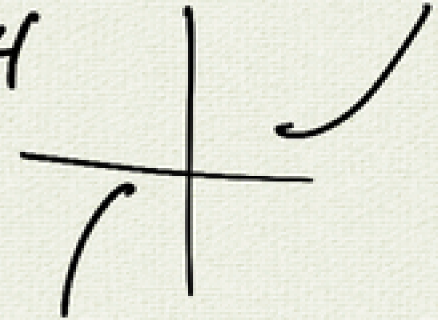
$$= \left(x^3 \qquad\qquad + 6\right)$$

$-2/3$

-4

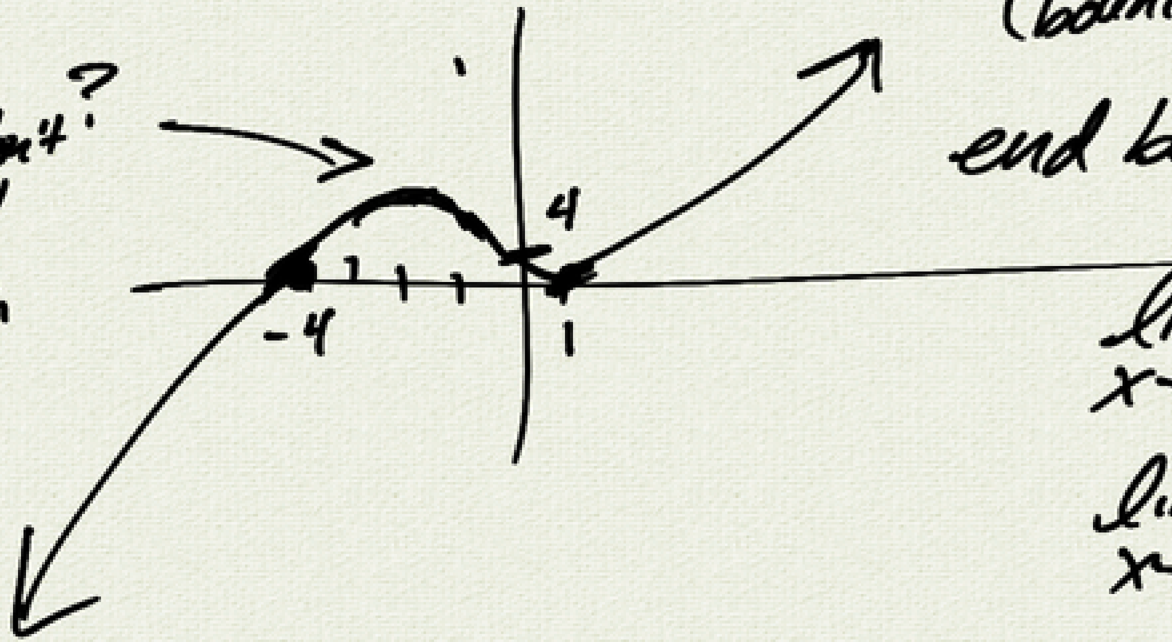
$$f(x) = -\frac{2}{3}(x+2)(x-1)(x-3)$$

(43) $g(x) = (x+4)(x-1)^2 = x^3 + \dots + 4$



multiplicity 2
(bounce)

we don't know how high?



end behavior like x^3:

$\lim\limits_{x\to\infty} g(x) = \infty$

$\lim\limits_{x\to-\infty} g(x) = -\infty$

3.6

15 $f(x) = 2x^3 + x^2 - 5x + 2$

one factor: $x+2 \Leftarrow f(-2) = 0$

find all zeros / factor completely

check: $f(-2) = 2(-8) + 4 + 10 + 2$
$= 0$ ✓

$$
\begin{array}{r|l}
 & 2x^2 - 3x + 1 \\
x+2 & 2x^3 + x^2 - 5x + 2 \\
 & \underline{2x^3 + 4x^2} \\
 & -3x^2 - 5x + 2 \\
 & \underline{-3x^2 - 6x} \\
 & x + 2 \\
 & \underline{x + 2} \\
 & 0 \Leftarrow \text{remainder}
\end{array}
$$

$f(x) = (x+2)(2x^2 - 3x + 1)$
$= (x+2)(2x-1)(x-1)$

zeros: $-2, \frac{1}{2}, 1$

Factor Thm

$x-a$ is a factor $\Longleftrightarrow$ a is a zero (root)

$x+2 \Longleftrightarrow -2$

$2x-1 \Longleftrightarrow \frac{1}{2}$

$x-1 \Longleftrightarrow 1$

(17) $f(x) = 2x^3 + 3x^2 + x + 6$

factor $x+2$

$\Longrightarrow f(-2) = 0$

check: $2(-8) + 3(4) + (-2) + 6$

$= -16 + 12 - 2 + 6$

$= 0$ ✓

4.4 More Polynomials

synthetic division

$f(x) = 2x^3 + 3x^2 + x + 6$

factor $x+2$

$x+2 \overline{)\,2x^3 + 3x^2 + x + 6}$

zero

$$\begin{array}{r|rrrr} \boxed{-2} & 2 & 3 & 1 & 6 \\ & & -4 & 2 & -6 \\ \hline & 2 & -1 & 3 & \boxed{0} \end{array}$$

coefficients: $2, 3, 1, 6$

$\times(-2)$

quotient: $2, -1, 3$ — remainder: 0

$2x^2 - x + 3$

$\Longrightarrow$

$f(x) = (x+2)(2x^2 - x + 3)$

$\dfrac{f(x)}{x+2} = 2x^2 - x + 3$

no remainder

$$x \cdot x^2 = x^3$$

$$x^m \cdot x^n = x^{m+n}$$

$$\underbrace{\underbrace{x \cdots x}_{m} \; \underbrace{x \cdots x}_{n}}_{m+n}$$

$$(x^m)^n = x^{mn}$$

$$\underbrace{\underbrace{x \cdots x}_{m} \; \underbrace{x \cdots x}_{m} \; \cdots \; \underbrace{x \cdots x}_{m}}_{n}$$

<u>example:</u>

$$f(x) = (x-1)(x-2)(x-3) + 1$$

$$= x^3 - 6x^2 + 11x - 5$$

<u>divide by $x-2$:</u> $\Leftarrow$ degree 1

$\rightarrow$ synthetic division

$$\begin{array}{r|rrrr} 2 & 1 & -6 & 11 & -5 \\ & & 2 & -8 & 6 \\ \hline & \underbrace{1 \quad -4 \quad 3}_{\text{quotient}} & & & \underbrace{\boxed{1}}_{\text{remainder}} \end{array}$$

$$f(x) = (x-2)(x^2 - 4x + 3) + 1$$

$$\frac{f(x)}{x-2} = x^2 - 4x + 3 + \frac{1}{x-2}$$

$\leftarrow$ remainder as a fraction

$$g(x) = (2x-3)(5x-7)$$

zeros: $\frac{3}{2}$, $\frac{7}{5}$ rational roots zeros

$$g(x) = 10x^2 - 29x + \underline{21}$$

Rational Roots Theorem

$g(x)$ polynomial (integer coeff.)

$$g(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_0$$

Suppose $\frac{p}{q}$ is rational zero.

Then $p | a_0$ and $q | a_n$.

Example: $h(x) = x^5 - 3x^4 - 5x^3 + 15x^2 + 4x - 12$

find all roots / factor completely

potential rational roots: $\dfrac{\pm 1, 2, 3, 4, 6, 12}{\pm 1}$

numerator

12 possibilities

$h(1) = 1 - 3 - 5 + 15 + 4 - 12$
$= 0$ ✓

$\Rightarrow x - 1$ is a factor

$$\begin{array}{r|rrrrrr} 1 & 1 & -3 & -5 & 15 & 4 & -12 \\ & & 1 & -2 & -7 & 8 & 12 \\ \hline & 1 & -2 & -7 & 8 & 12 & \boxed{0} \end{array}$$

remainder

$x^4 - 2x^3 - 7x^2 + 8x + 12$

$h(x) = (x-1)(x^4 - 2x^3 - 7x^2 + 8x + 12)$

try -1: $(-1)^4 + 2 - 7 - 8 + 12 = 0$ ✓

$$\begin{array}{r|rrrrr} -1 & 1 & -2 & -7 & 8 & 12 \\ & & -1 & 3 & 4 & -12 \\ \hline & 1 & -3 & -4 & 12 & \boxed{0} \end{array}$$

$h(x) = (x^3 - 3x^2 - 4x + 12)(x+1)(x-1)$

guess + check 2: $h(2) = \underbrace{(8 - 12 - 8 + 12)}_{0}\underline{(3)(1)}$
$= 0$ ✓

don't care

$$\begin{array}{r|rrrr} 2 & 1 & -3 & -4 & 12 \\ & & 2 & -2 & -12 \\ \hline & 1 & -1 & -6 & \boxed{0} \end{array}$$

$x^2 - x - 6$

$(x-3)(x+2)$

$\Rightarrow h(x) = (x+1)(x-1)(x-2)(x-3)(x+2)$

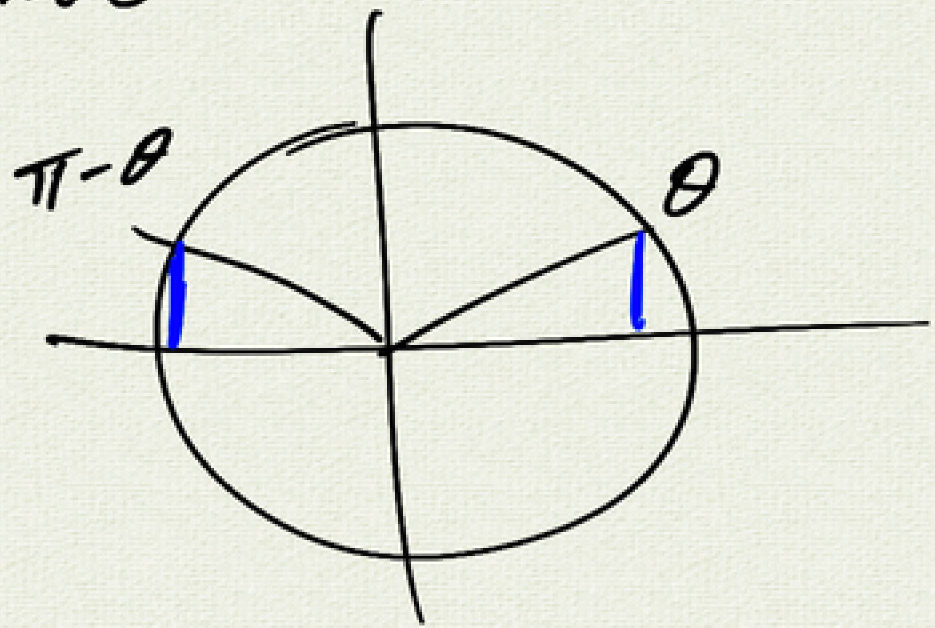
complete factorization

all zeros: $\pm 1, \pm 2, 3$

$$\sin(\pi-\theta)=\sin\theta$$

sum formula

$$\underbrace{\sin\pi}_{0}\cos\theta-\underbrace{\cos\pi}_{-1}\sin\theta$$



$$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$$