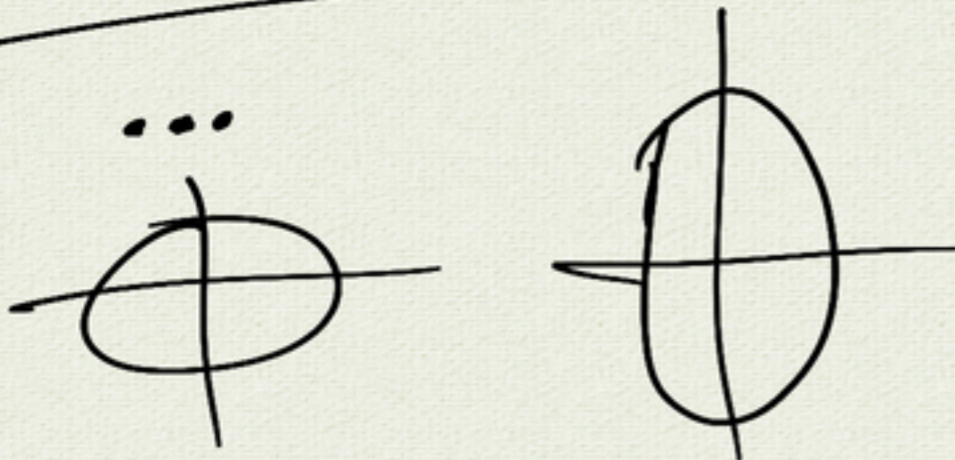
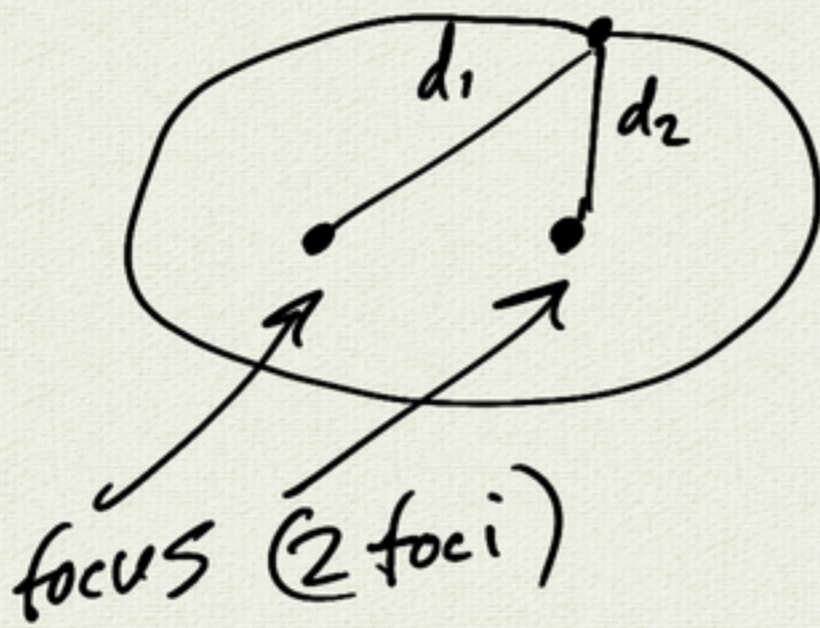


5.2 Ellipses



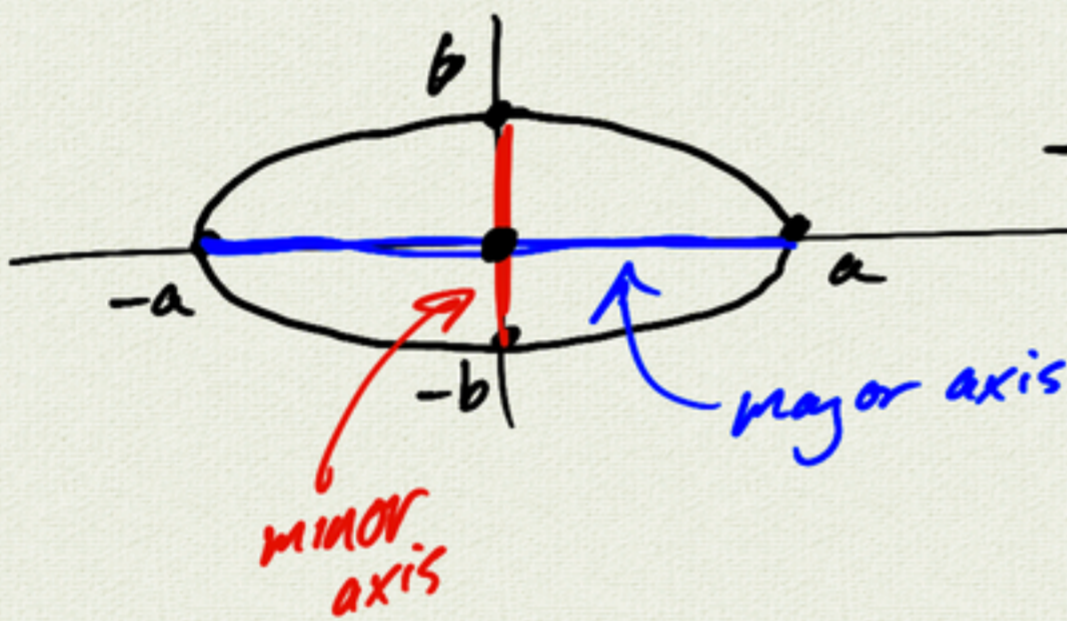
ellipse: set of points where $d_1 + d_2 = \text{const}$



more precise:

equation: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($b < a$)

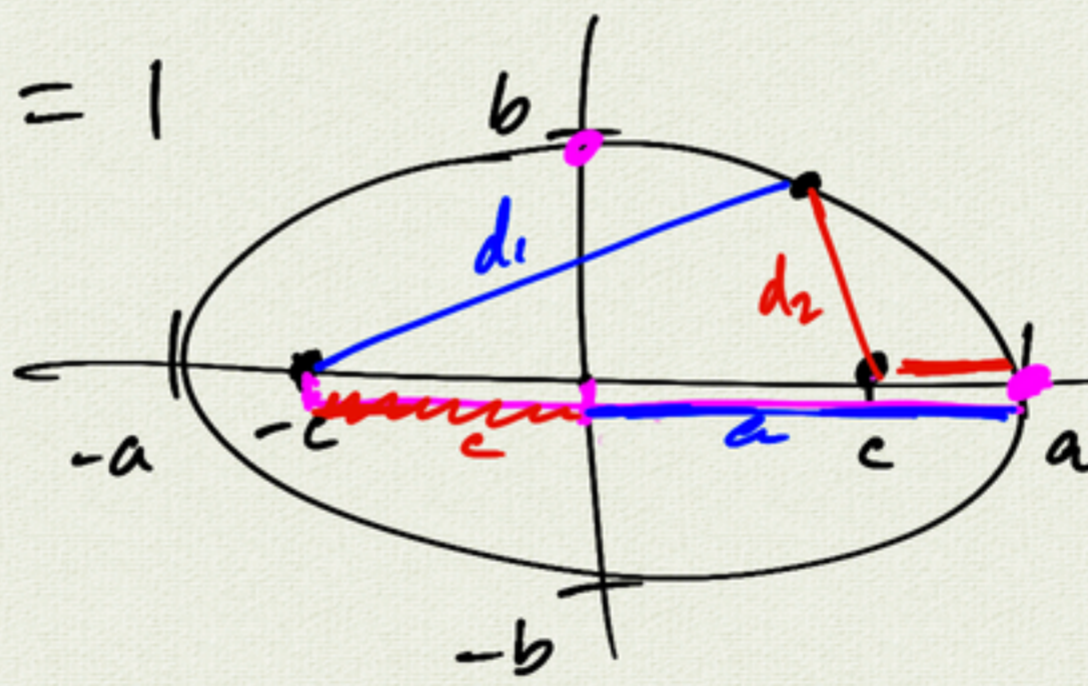
$y=0 \Rightarrow \frac{x^2}{a^2} = 1$
 $x^2 = a^2$
 $x = \pm a$



center (h, k) :
 $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$a=b$
 $\frac{x^2}{a^2} + \frac{y^2}{a^2} = 1$
 $x^2 + y^2 = a^2$
 $(x-h)^2 + (y-k)^2 = a^2$
 \rightarrow center (h, k)

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

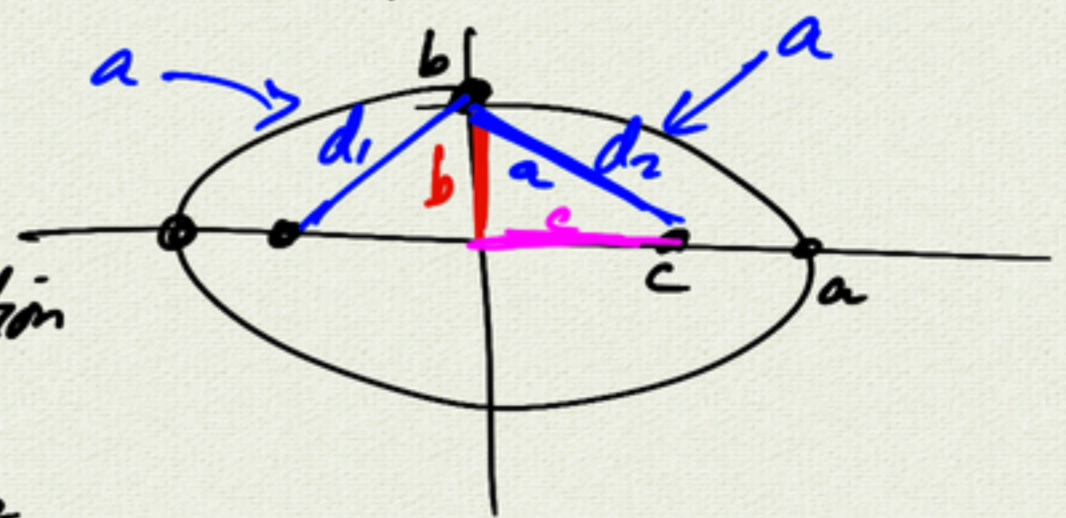


$d_1 + d_2 = \text{const}$
 at any point on the ellipse

consider $(a, 0)$:
 $d_1 + d_2 = (a+c) + (a-c)$
 $= 2a$ (=const.)

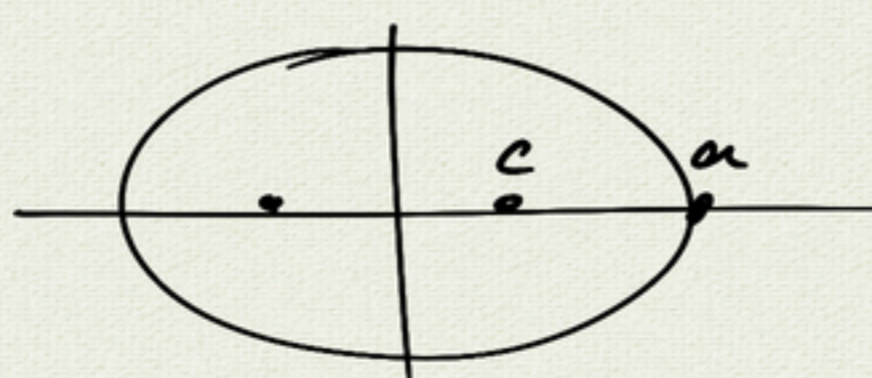
consider $(0, b)$:

\Rightarrow Pythagorean relation
 $b^2 + c^2 = a^2$
 $c^2 = a^2 - b^2$



$e = \frac{c}{a}$ eccentricity

$c=0$ circle
 $e=0$



$c \rightarrow a$
 $e \rightarrow 1$ (more oblong)

$0 \leq e < 1$ ellipse

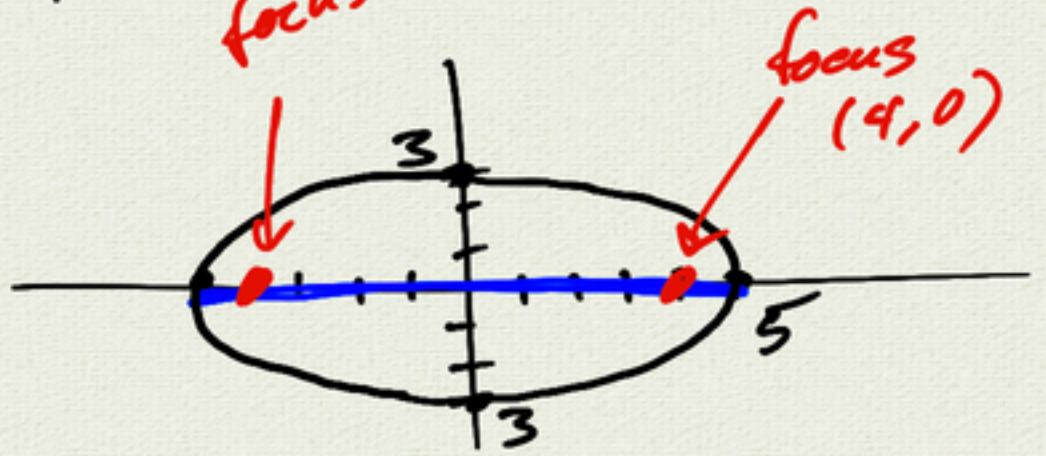
example:

$$\frac{x^2}{25} + \frac{y^2}{9} = 1$$

$$a=5$$
$$b=3$$

$$c^2 = a^2 - b^2$$
$$= 25 - 9$$
$$= 16$$
$$c = 4$$

$$e = \frac{c}{a} = \frac{4}{5}$$

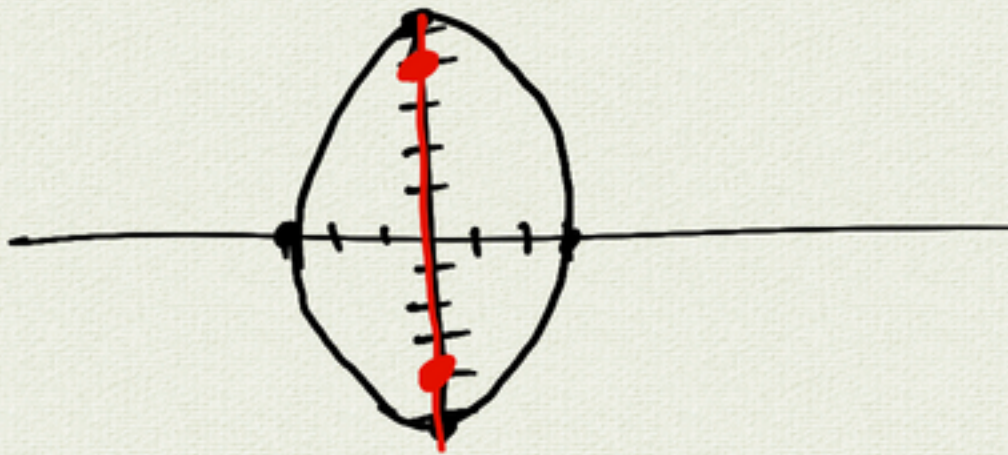


example 2

$$\frac{x^2}{9} + \frac{y^2}{25} = 1$$

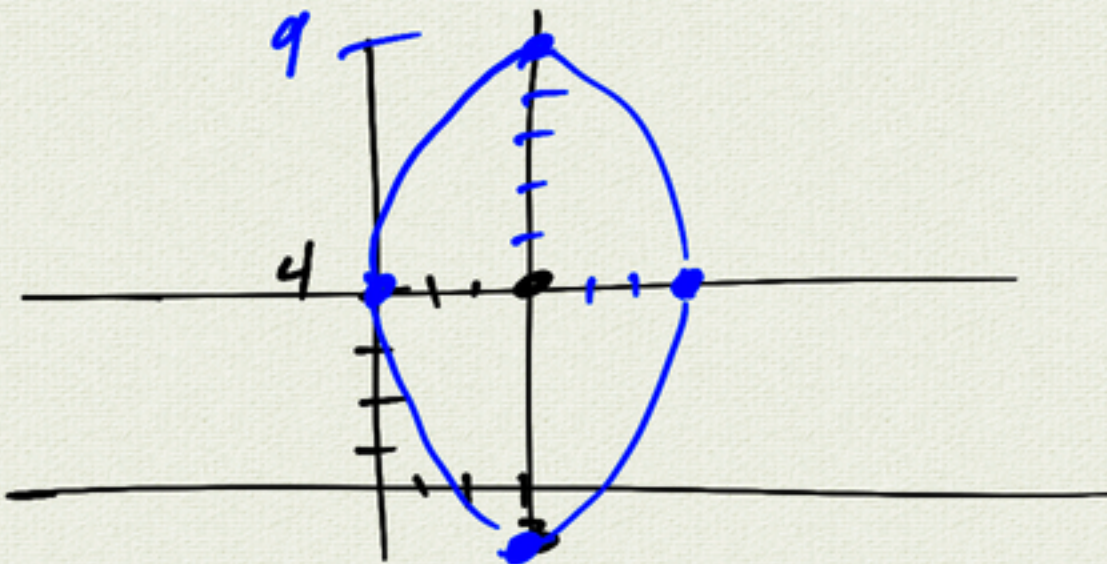
$$c^2 = a^2 - b^2$$

↑
bigger



example 3 center (3, 4)

$$\frac{(x-3)^2}{9} + \frac{(y-4)^2}{25} = 1$$



example 4

$$x^2 + y^2 - 2x - 4y - 31 = 0$$

$$(x^2 - 2x + 1) + (y^2 - 4y + 4) = 31$$

+1

+4

$$(x-1)^2 + (y-2)^2 = 36$$

circle

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$(x+a)^2 = x^2 + 2ax + a^2$$

$$\frac{x^2}{4} + \frac{y^2}{9} = 2 \Rightarrow \frac{x^2}{8} + \frac{y^2}{18} = 1$$

\nearrow
 $\times 2$

$a \rightarrow a \cdot \sqrt{2}$ scale