5.3 Hyperbolas

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

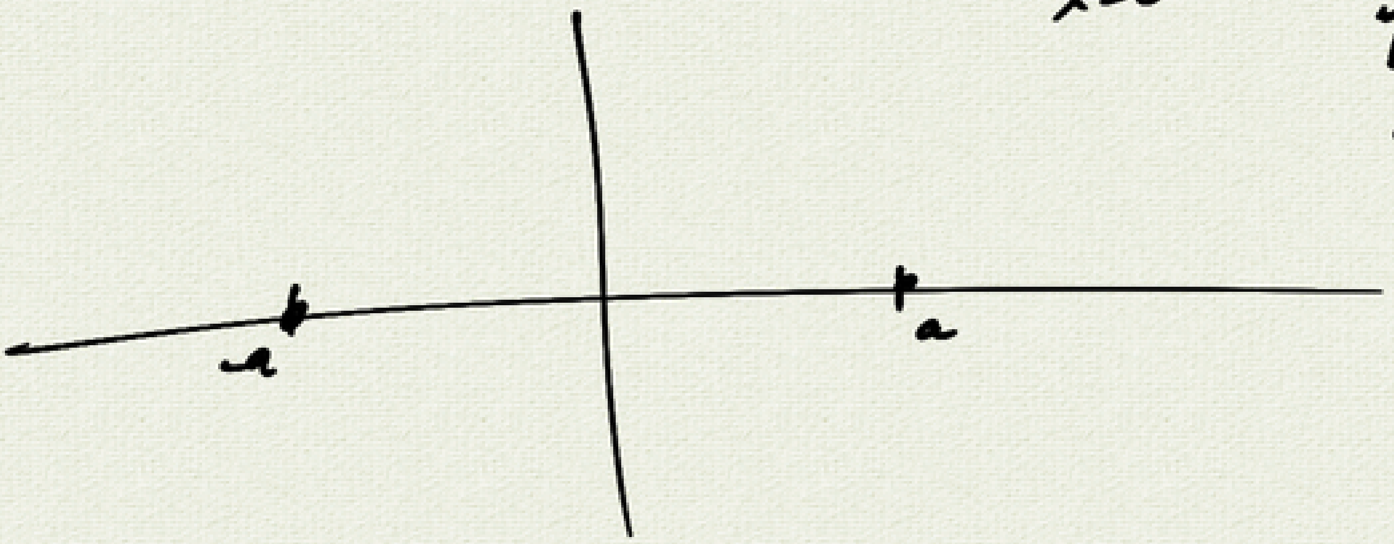
$y=0 \Rightarrow \frac{x^2}{a^2} = 1$

$x^2 = a^2$

$x = \pm a$

$x=0 \Rightarrow -\frac{y^2}{b^2} = 1$

$y^2 = -b^2\,?$



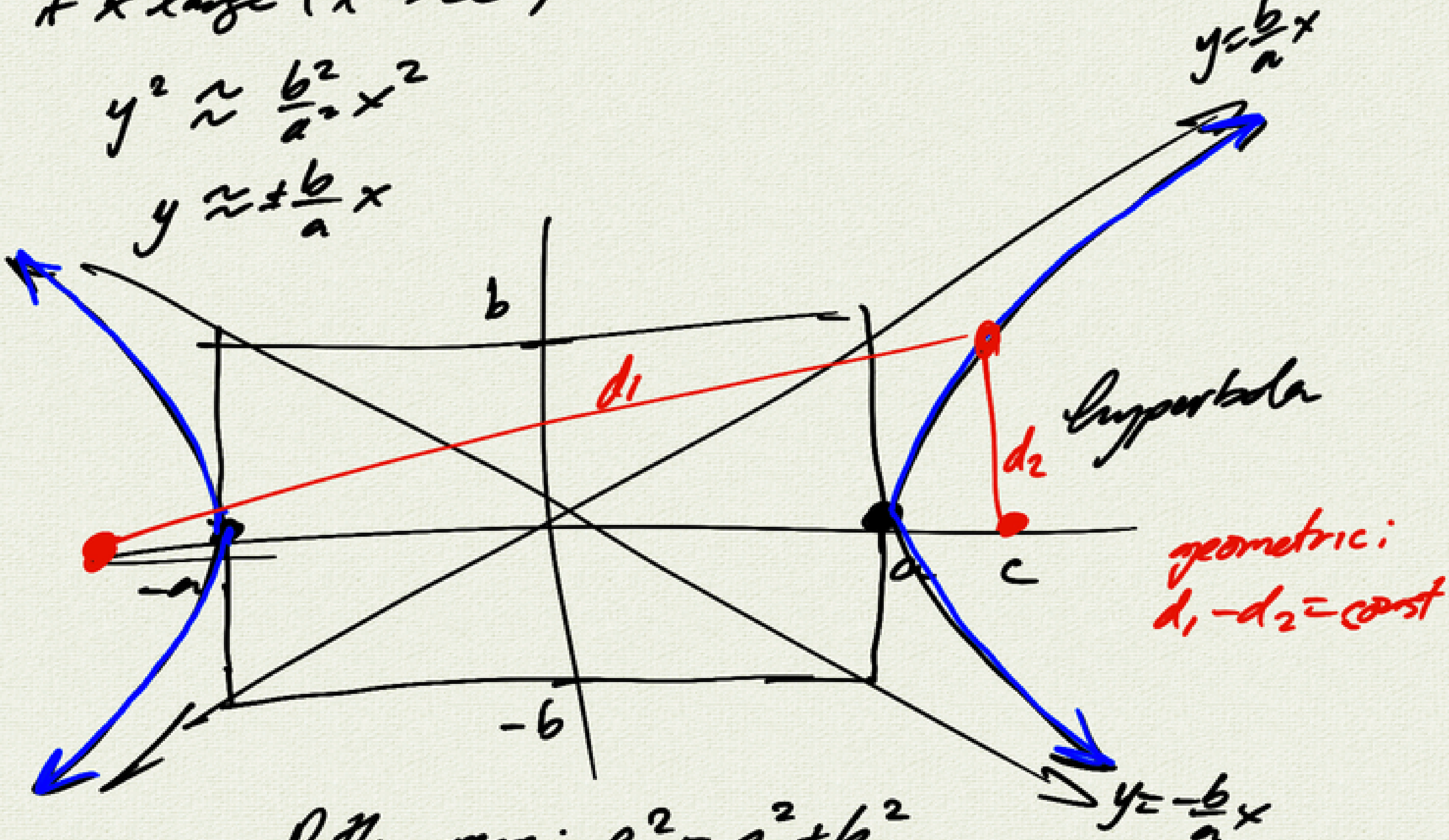
$$\frac{y^2}{b^2} = \frac{x^2}{a^2} - 1$$

$$y^2 = \frac{b^2}{a^2}x^2 - 1$$

if x large ($x \to \infty$)

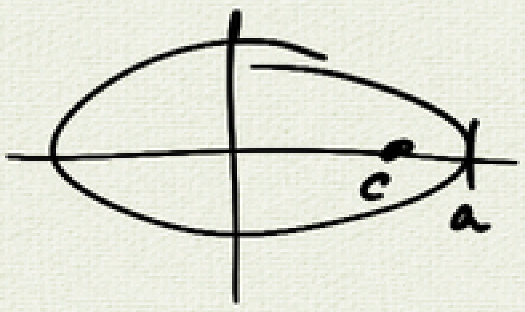
$$y^2 \approx \frac{b^2}{a^2}x^2$$

$$y \approx \pm\frac{b}{a}x$$



Pythagorean relation: $c^2 = a^2 + b^2$

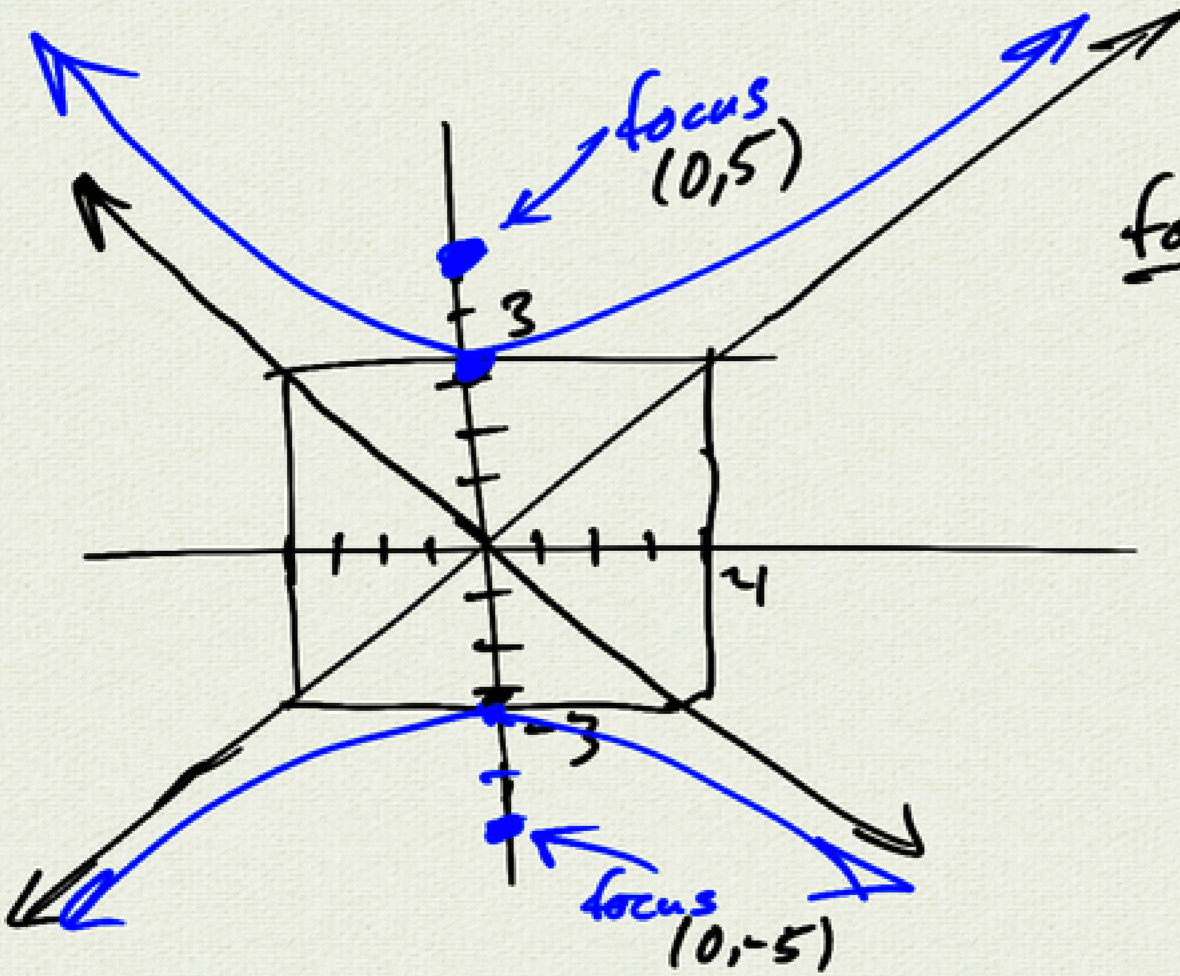
eccentricity $= e = \frac{c}{a} > 1$



$e=0$ circle
$0<e<1$ ellipse
$e=1$ parabola
$e>1$ hyperbola

example: $\frac{y^2}{9} - \frac{x^2}{16} = 1$

center (h,k): $\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$



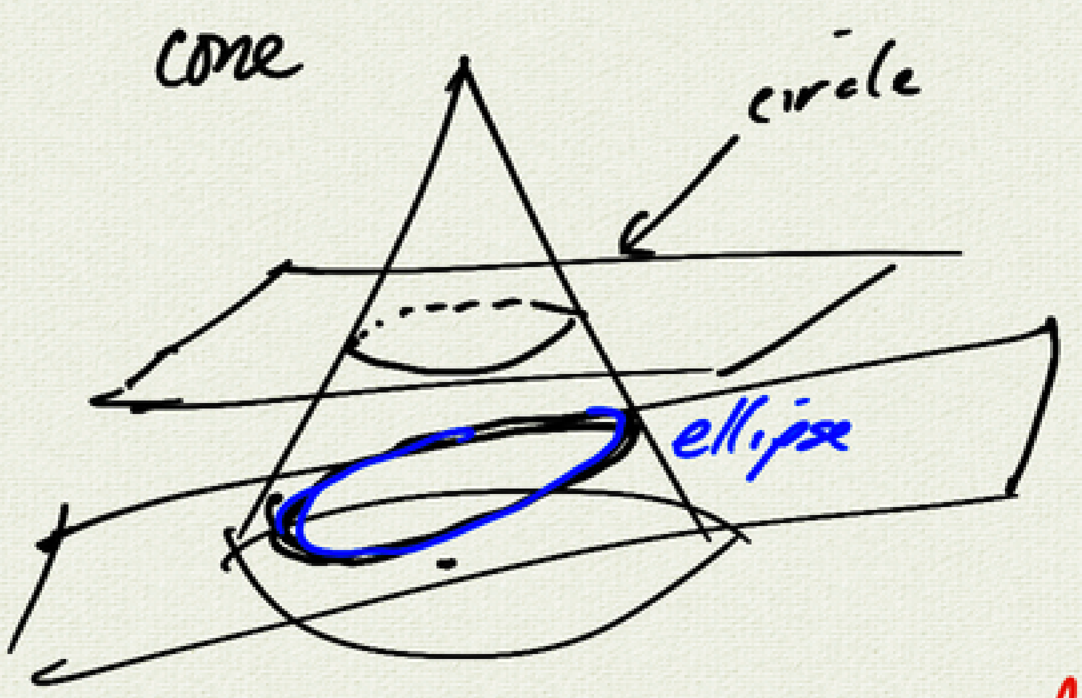
foci: $c^2 = a^2 + b^2$

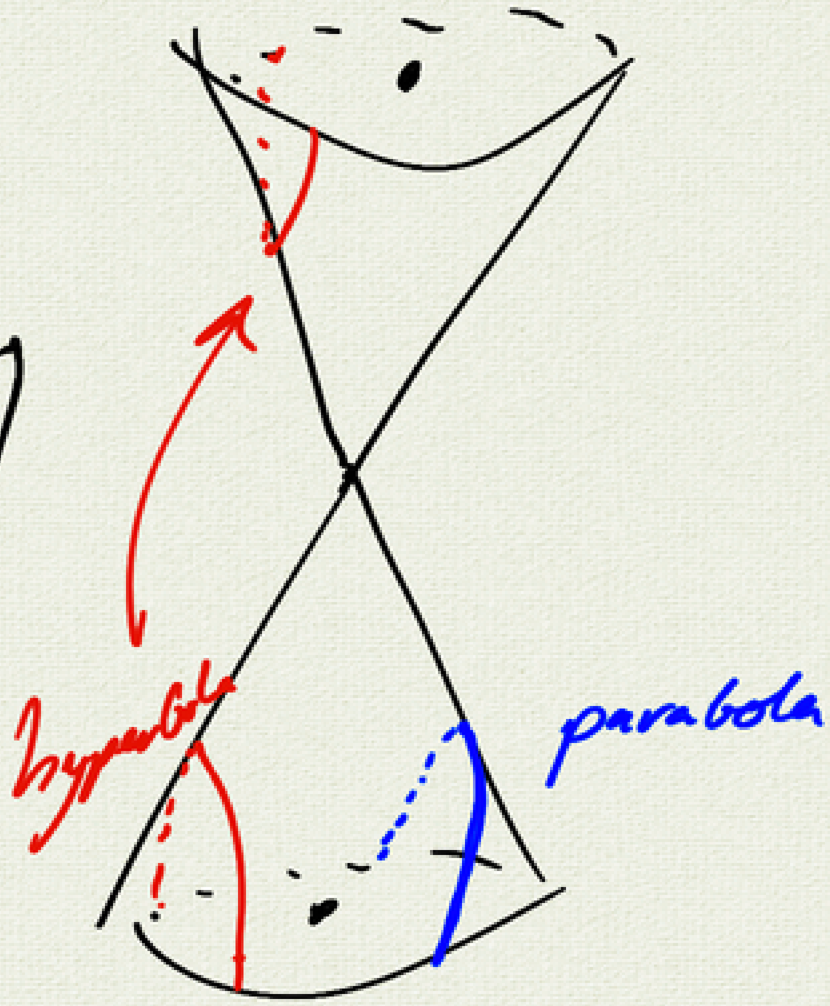
$= 9 + 16$

$= 25$

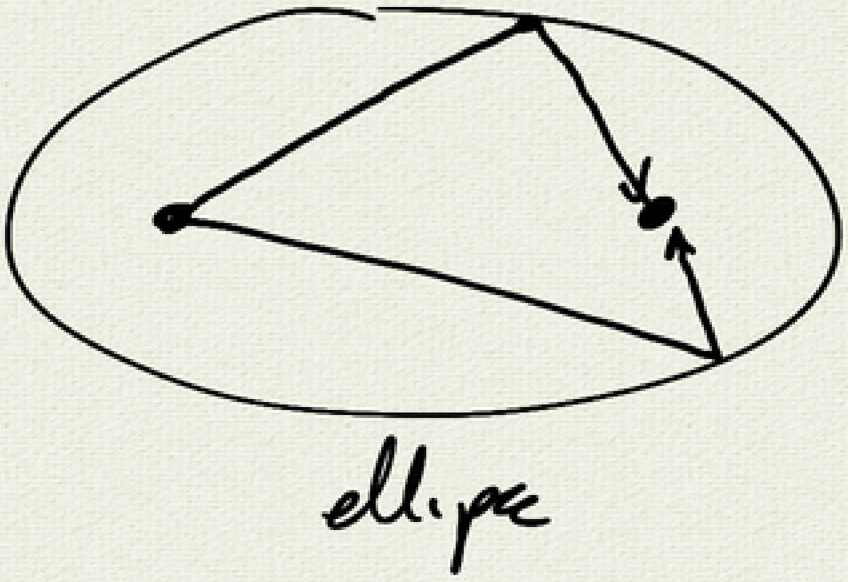
$c = 5$

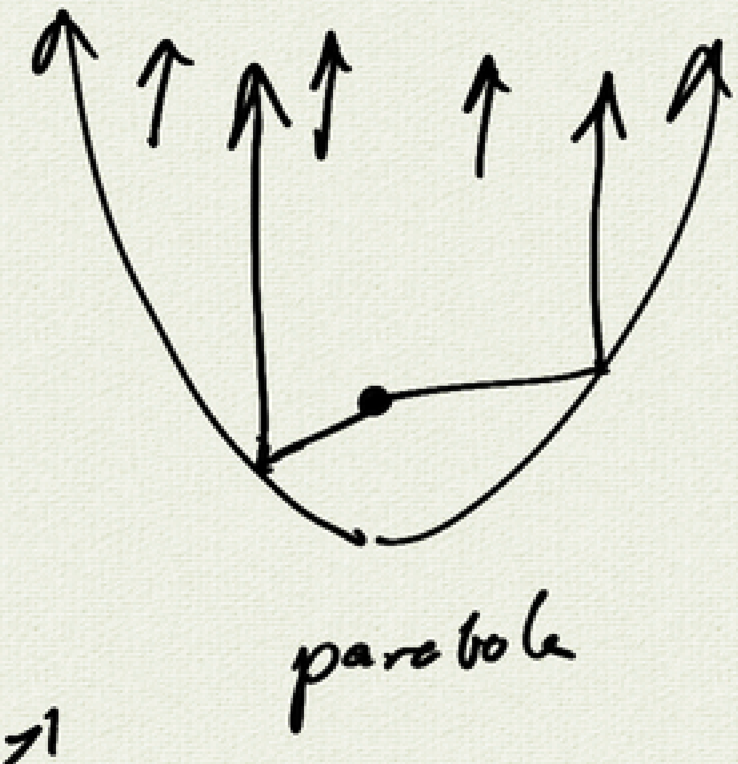
conic sections



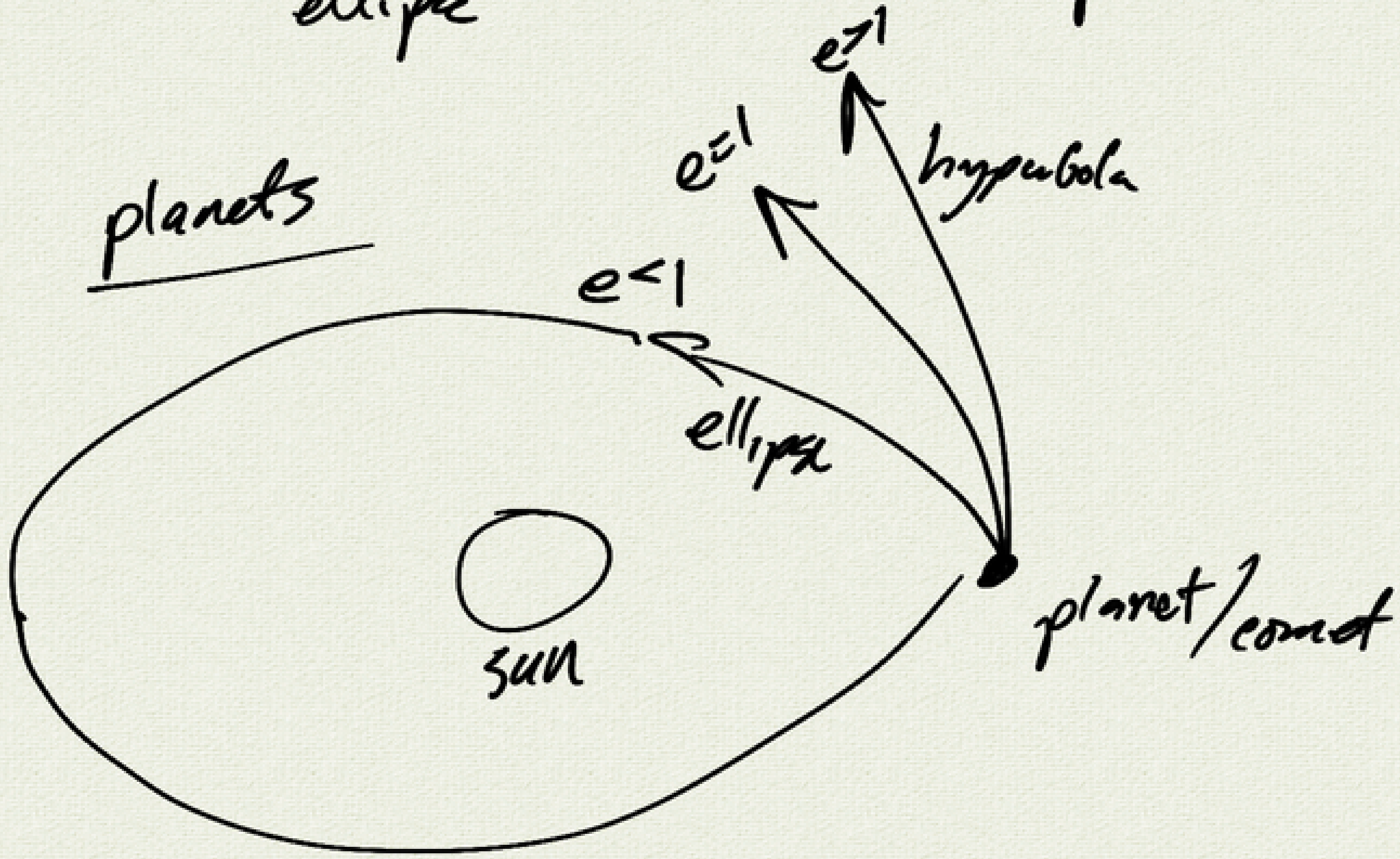


reflective properties



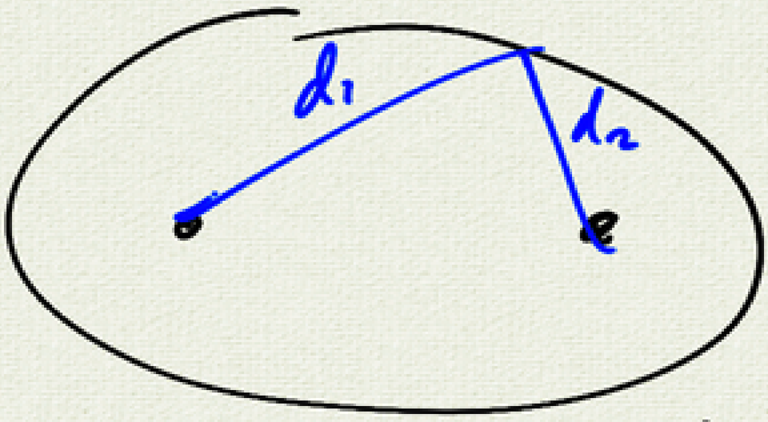


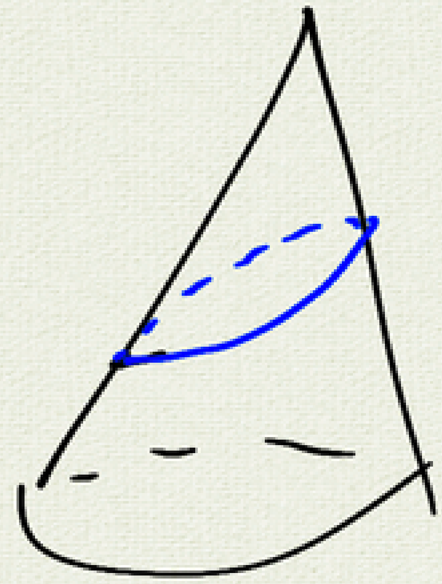
planets



ellipses

2 geometric definitions:





$d_1 + d_2 = \text{const}$

slice of cone

Dandelin