6.1 Combinatorics

discrete mathematics ⟷ continuous

combinatorics = counting

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example:
Chipotle (simple version)

bowl / burrito / taco	beef / chicken / tofu	mild / med / hot	cheese / no	corn / no	lettuce / no
3	3	3	2	2	2

possible orders = $3 \cdot 3 \cdot 3 \cdot 2 \cdot 2$

(multiply ⟷ independent choices)

... big tree

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pizza restaurant: 10 toppings.
how many (different) pizzas?

pepperoni / no	bell pepper / no	...	mushroom / no

$2 \cdot 2 \cdot \dots \cdot 2$ (2^{10} possible)

binary sequences of length 10

$$\begin{matrix} 000\dots0 \\ 000\dots01 \\ \vdots \\ 111\dots1 \end{matrix}$$

$0\ 0\ \bullet \quad \bullet$
$1\ 1\ 1 \dots 1$

2^{10}

# of subsets of the set of toppings (10)

$\emptyset = \{\}$ empty

{pepperoni}

{pepperoni, bell pepper}

Summary:
pizzas w/ 10 toppings $= 2^{10}$

$=$ # binary sequences of length 10

$=$ # of subsets of a set of size 10

$\emptyset \longleftrightarrow 000\dots0$

{pepperoni} $\longleftrightarrow 100\dots0$

{pepperoni, bell pepper} $\longleftrightarrow 110\dots0$

$\vdots$

supreme (all toppings) $\longleftrightarrow 111\dots1$

permutation = reordering

example $\{A, B, C\}$

ABC BAC CAB
ACB BCA CBA

] 6 permutations

$3 \cdot 2 \cdot 1 = 6$

← 3! "3 factorial"

$\{A, B, C, D\}$

how many permutations?

$4 \cdot 3 \cdot 2 \cdot 1$

$= 24$

$4!$ "4 factorial"

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example: class of 20

pick president, VP, treasurer.

how many ways?

$\frac{20}{P} \cdot \frac{19}{VP} \cdot \frac{18}{T}$ (calculator)

← $_{20}P_3$

← # of choices

← how many you're ordering

combinations

$\{A, B, C, D\}$

ways to pick 2 (order does not matter)

AB BC CD
AC BD
AD

$$\#\text{ways} = \frac{4 \cdot 3}{2} = 6$$

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ways to pick 3 from $\{A, B, C, D\}$

$$\frac{4 \cdot 3 \cdot 2}{3!} = 4$$

notation:

$${}_4C_3 = \binom{4}{3} = \begin{matrix}\#\text{ ways to choose} \\ 3 \text{ items from } 4\end{matrix}$$

$$\boxed{\binom{4}{3} = \binom{4}{1}}$$

$\binom{4}{0}$ = # of ways to choose nothing $= 1$

$\binom{4}{1}$ = # of ways to pick 1 thing from 4 $= 4$

$\binom{n}{0} = 1$
$\binom{n}{1} = n$
$\vdots$
$\binom{n}{n-1} = n$
$\binom{n}{n} = 1$

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example: dinner party 12 people

cheers: # of clinks = ?

$$\hookrightarrow = \binom{12}{2} = \begin{matrix}\#\text{ of ways to} \\ \text{pick 2 from 12}\end{matrix}$$

$$\frac{12 \cdot 11}{2} = 66$$