practice:

$\binom{10}{2}$ = "10 choose 2" = $_{10}C_2$

= # ways to pick 2 items from 10

$= \frac{10 \cdot 9}{2}$ ← when order doesn't matter

= 45

permutation = reordering (order matters)

subsets of $\{A, B, C\}$ (# subsets = $2^3 = 8$)

$\emptyset$ 000	$\{A\}$ 100	$\{A,B\}$ 110	$\{A,B,C\}$ 111
	$\{B\}$ 010	$\{B,C\}$ 011	
	$\{C\}$ 001	$\{A,C\}$ 101	

0/1 0/1 0/1

$\binom{3}{0}=1$ $\binom{3}{1}=3$ $\binom{3}{2}=3$ $\binom{3}{3}=1$

$\Rightarrow \binom{3}{0}+\binom{3}{1}+\binom{3}{2}+\binom{3}{3} = 2^3$

$$\binom{n}{r} = \text{\# ways to pick } r \text{ items from } n$$

$$= \frac{\overbrace{n(n-1)(n-2)\dots(n-r+1)}^{r \text{ items}}}{r!}$$

$r!$ ← # ways to reorder r items

$$= \frac{n(n-1)(n-2)\dots(n-r+1)}{r!} \frac{(n-r)(n-r-1)\dots(1)}{(n-r)(n-r-1)\dots(1)}$$

$$\binom{n}{r} = \frac{n!}{r!(n-r)!}$$

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example:

$$\binom{100}{2} = \frac{100!}{2!\,98!}$$

too big

$$= \frac{100 \cdot 99}{2} \quad \text{ok}$$

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permutations: $_3P_3$ = # ways to order 3 items

$= 3!$

$= \underline{3} \cdot \underline{2} \cdot \underline{1}$

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example: # ways to pick P, VP from 100 people

$\underline{100} \cdot \underline{99}$

$\{A, C, G, T\}$

codon: $\underline{4} \cdot \underline{4} \cdot \underline{4} = 64$

polynomial multiplication:

Pascal's Triangle

$$(x+y)^0 = 1$$
$$(x+y)^1 = x+y$$
$$(x+y)^2 = x^2 + 2xy + y^2$$
$$(x+y)^3 = (x+y)(x^2+2xy+y^2)$$
$$= x^3 + 2x^2y + xy^2$$
$$x^2y + 2xy^2 + y^3$$
$$= x^3 + 3x^2y + 3xy^2 + y^3$$

$$\begin{array}{ccccccccc} & & & & 1 & & & & \\ & & & 1 & & 1 & & & \\ & & 1 & & 2 & & 1 & & \\ & 1 & & 3 & & 3 & & 1 & \\ 1 & & 4 & & 6 & & 4 & & 1 \end{array}$$

$$\begin{array}{ccccccc} & & & \binom{0}{0} & & & \\ & & \binom{1}{0} & & \binom{1}{1} & & \\ & \binom{2}{0} & & \binom{2}{1} & & \binom{2}{2} & \\ \binom{3}{0} & & \binom{3}{1} & & \binom{3}{2} & & \binom{3}{3} \end{array}$$

$$\Longrightarrow (x+y)^4 = x^4 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + y^4$$

Binomial Theorem:

$$(x+y)^n = \binom{n}{0}x^ny^0 + \binom{n}{1}x^{n-1}y^1 + \dots + \boxed{\binom{n}{k}x^{n-k}y^k} + \dots + \binom{n}{n}x^0y^n$$

WTF?

why this formula?

questions:

1. why $\binom{n}{k}$?

2. why the triangle?
(what is recursive about $\binom{n}{k}$?)

① why $\binom{n}{k}$?

$$(x+y)^3 = (\underline{x}+y)(\underline{x}+y)(\underline{x}+y)$$

$$= \binom{3}{0}x^3y^0 + \binom{3}{1}x^2y^1 + \dots$$

ways to choose 0 y's

ways to choose 1 y out of the 3 factors

in general:

$(x+y)^n$: $\binom{n}{k}x^{n-k}y^k$

ways to choose k y's from n factors

$$= \binom{n}{n-k}$$

examples:

① expand binomial

$$(2x-y)^5$$

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & \boxed{5} & & 10 & & 10 & & 5 & & 1
\end{array}
$$

$$\binom{n}{0} \quad \binom{n}{1} = n$$

$$=$$

$$(2x)^5 + 5(2x)^4(-y)^1 + 10(2x)^3\underline{(-y)^2} + 10(2x)^2(-y)^3 + 5(2x)^1(-y)^4 + (-y)^5$$

$$= 32x^5 - 80x^4y + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5$$

② find $\underbrace{x^7y^5}$ term in $(-x+2y)^{12}$

$$= \binom{12}{5}(-x)^7\underline{(2y)^5}$$

$$= \binom{12}{7}$$

y's we choose out of 12

$$\binom{n}{k} = \binom{n}{n-k}$$

$$\binom{12}{5} = \frac{\cancel{12}\cdot 11\cdot \cancel{10}\cdot 9\cdot 8}{\cancel{5}\cdot\cancel{4}\cdot\cancel{3}\cdot\cancel{2}\cdot 1} = 11\cdot 9\cdot 8$$

1

1 1

1 2 1

1 3 3 1

$$\binom{0}{0}$$

$$\binom{1}{0} \quad \binom{1}{1}$$

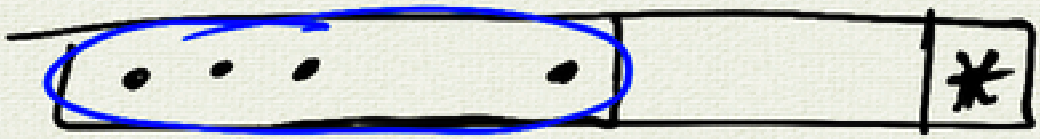
$$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$$

$$\binom{3}{0} \quad \boxed{\binom{3}{1}} \quad \binom{3}{2} \quad \binom{3}{3}$$

$$\binom{n}{k}$$

$$\boxed{\binom{n}{k} \stackrel{?}{=} \binom{n-1}{k-1} + \binom{n-1}{k}}$$

n items



choose k

with star

no star

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

binomial coefficients

combinatorics