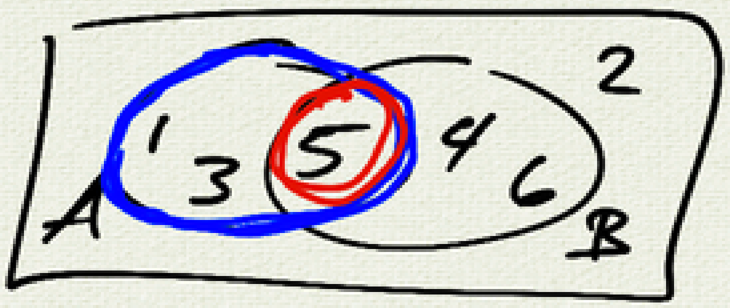
6.4 More Probability

conditional probability

rolling 1 die   outcomes $\{1,2,3,4,5,6\}$



$A = \{odd\} = \{1,3,5\}$ $P(A) = \frac{3}{6} = \frac{1}{2}$

$B = \{big\} = \{4,5,6\}$ $P(B) = \frac{1}{2}$

A, B independent $\Leftrightarrow$ $P(A \cap B) = P(A)\,P(B)$

$$\frac{1}{6} \stackrel{?}{=} \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)$$

not independent

$$\boxed{P(B|A) = \frac{P(B \cap A)}{P(A)}}$$

definition

"probability of B given A"
conditional probability

A, B independent $\Leftrightarrow$ $P(A \cap B) = P(A)\,P(B)$

$$\frac{P(B \cap A)}{P(A)} = P(B)$$

$A \cap B = B \cap A = \{5\}$

$$P(B|A) = P(B)$$

$$\frac{1}{3} = \frac{1/6}{1/2} = \frac{P(B \cap A)}{P(A)} \stackrel{?}{=} \frac{1}{2}$$

$\Longrightarrow$ A, B (in this case) are not independent coin flipping (not necessarily fair)

coin biased: $P(\text{heads}) = .9$

$P(\text{tails}) = .1$

flip 3 times: _ _ _

$P(1\ 1\ 1) = (.9)(.9)(.9) = (.9)^3 \approx .7$

heads

0 = tails

$P(1\ 1\ 0) = \boxed{(.9)(.9)(.1)}$

$P(1\ 0\ 1) = (.9)(.1)(.9)$

$P(0\ 1\ 1) = (.1)(.9)(.9)$

$P\left(\begin{matrix}2 \text{ heads}\\ 1 \text{ tails}\end{matrix}\right) = 3(.9)^2(.1)^1$

$\binom{3}{1}$ or $\binom{3}{2}$

$P\left(\begin{matrix}2 \text{ heads}\\ 1 \text{ tail}\end{matrix}\right) = \boxed{}\ \boxed{.9\ .9\ .1} = \binom{3}{1}(.9)^2(.1)^1$

$= \binom{3}{2}(.9)^2(.1)^1$

how many ways to pick 1 out of 3 spots to be the ".1"

$P(\text{at least 2 heads}) = P(3 \text{ heads}) + P(2 \text{ heads})$

$= \binom{3}{3} \underline{.9}\ \underline{.9}\ \underline{.9} + \underline{\qquad}$

$= \binom{3}{3}(.9)^3(.1)^0 + \binom{3}{2}(.9)^2(.1)^1$

binomial distribution

free throws

$p = .7$

10 throws

what is $P(7 \text{ successes})$?

$P(7) = \binom{10}{7}(.7)^7(.3)^3$

${}_{10}C_{7}$ (nCr)

$P(10) = (.7)^{10} = \binom{10}{0}(.7)^{10}(.3)^0$

p = success probability

$1 = p + (1-p)$

$1^n = (p + (1-p))^n$

$= p^n + \binom{n}{1}p^{n-1}(1-p)^1 + \binom{n}{2}p^{n-2}(1-p)^2 + \dots$

$P(\text{all success})$

$P\left(\begin{matrix}n-1 \text{ success}\\ 1 \text{ failure}\end{matrix}\right)$

Jar M&M's
20 good
10 bad

grab 7

$$P(5 \text{ good} \; (2 \text{ bad})) = \frac{\binom{20}{5}\cdot\binom{10}{2}}{\binom{30}{7}}$$

hypergeometric

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47 - 49

12 blue
36 not blue

grab 5

(47) $$P(\text{all blue}) = \frac{\binom{12}{5}\cancel{\binom{36}{0}}}{\binom{48}{5}} = \frac{\frac{12\cdot 11\cdot 10\cdot 9\cdot 8}{5!}}{\frac{48\cdot 47\cdot 46\cdot 45\cdot 44}{5!}}$$

$$= \frac{12}{48}\cdot\frac{11}{47}\cdot\frac{10}{46}\cdot\frac{9}{45}\cdot\frac{8}{44}$$

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(49) $$P(3 \text{ blue} \; 2 \text{ non blue}) = \frac{\binom{12}{3}\binom{36}{2}}{\binom{48}{5}} = \boxed{\binom{5}{2}}\frac{12\cdot 11\cdot 10\cdot 36\cdot 35}{48\cdot 47\cdot 46\cdot 45\cdot 44}$$

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binomial: P(success) = p (infinite)
choose n, calculate P(k successes)

hypergeometric: N successful, M failure (finite)
choose n, calculate P(k success)