

(53)

20 numbers picked

80 cards

20 selected

win if: 3, 4, 5 or 20 match

$$P(\text{5 matches selected}) = \frac{\binom{20}{5} \binom{60}{15}}{\binom{80}{20}}$$

15 not selected

hypergeometric

$$P(\geq 5 \text{ matches}) = P(5) + P(6) + P(7) + \dots + P(20)$$

## 6.5 Sequences

1, 2, 3, 4, 5, 6, ...

1, 3, 5, 7, ...

2, 4, 6, 8, ...

1, 1, 1, 1, ...

1,  $\frac{1}{2}$ ,  $\frac{1}{4}$ ,  $\frac{1}{8}$ , ...

0, 1, 0, 1, 0, 1, ... no limit

}  $\rightarrow \infty$

$\rightarrow 1$

$\rightarrow 0$

notation:

$a_1, a_2, a_3, a_4, \dots, a_n$

$\uparrow$   $n^{\text{th}}$  term

$\{a_n\}$

$\{a_n\}_{n=1}^{\infty}$

example:

$$a_n = 2n$$

2, 4, 6, 8, 10, ...

$n$	$a_n$
1	2
2	4
3	6

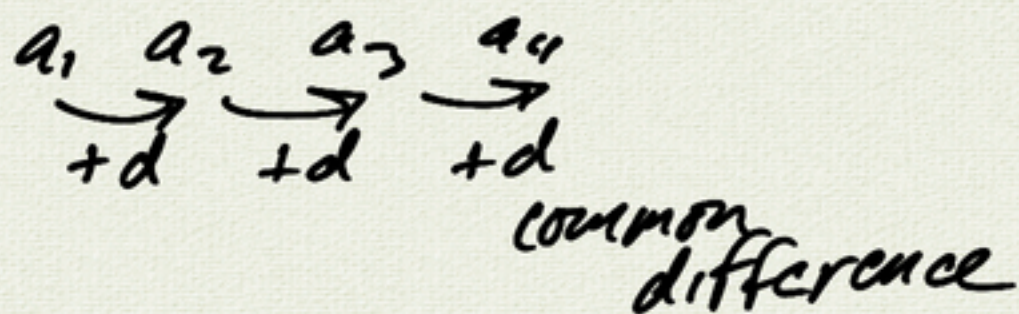
explicit formula

$$a_n = \left(\frac{1}{2}\right)^n$$

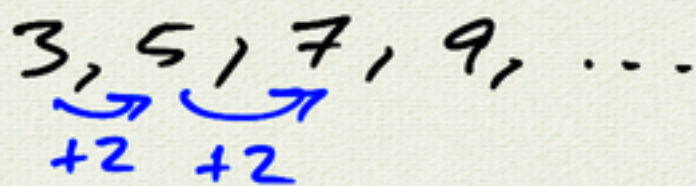
$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \dots$

$$a_{n+1} = \frac{1}{2} \cdot a_n \leftarrow \text{recursive formula}$$

# arithmetic sequences



Example:



common difference = 2

recursive formula:

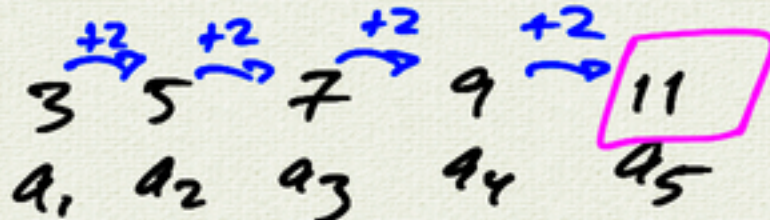
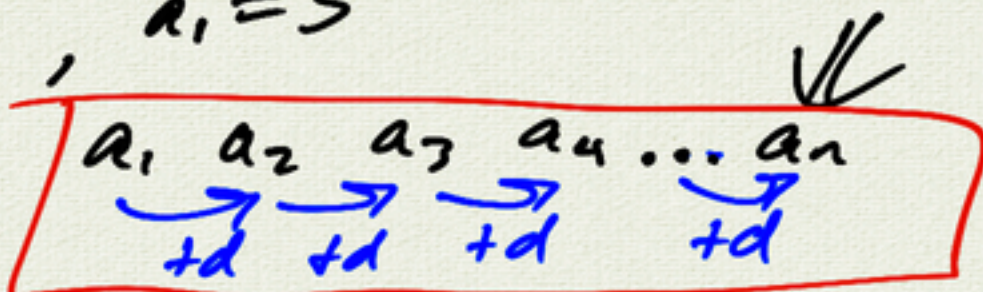
$$a_{n+1} = (a_n) + 2, \quad a_1 = 3$$

explicit formula

$$a_n = a_1 + (n-1)d$$

$$a_n = 3 + (n-1)2$$

$$3, 5, 7, 9, \dots$$



$$a_5 = a_1 + \underbrace{(5-1)}_4 \cdot \underbrace{2}_d$$

$$= 11$$

Example:

$$\begin{aligned} a_{101} &= 3 + (101-1) \cdot 2 \\ &= 203 \end{aligned}$$

example: arithmetic sequence

$$a_3 = 6 \quad a_8 = 21$$

find the explicit formula (and the first few terms)

$$\begin{array}{cccccccc} & & 6 & & & & 21 & \\ \hline a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & a_8 \end{array}$$

$$a_n = a_1 + (n-1)d$$

$$6 = a_3 = a_1 + 2d$$

$$21 = a_8 = a_1 + 7d$$

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$$15 = 5d$$

$$\Rightarrow d = 3$$

$$6 = a_1 + 2 \cdot 3$$

$$\Rightarrow a_1 = 0$$

$$a_n = 0 + (n-1)3$$

$$\boxed{a_n = 3n - 3} \text{ explicit formula}$$

0, 3, 6, 9, ...

# geometric sequence

$$\begin{array}{cccc} a_1 & a_2 & a_3 & a_4 \\ \xrightarrow{\cdot r} & \xrightarrow{\cdot r} & \xrightarrow{\cdot r} & \\ \text{common} & & & \\ \text{ratio} & & & \end{array}$$

example:  $1, 2, 4, 8, 16, 32, \dots$

$$\xrightarrow{\cdot 2} \xrightarrow{\cdot 2} \xrightarrow{\cdot 2}$$

$$1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \dots$$
$$\xrightarrow{\cdot \frac{1}{3}} \xrightarrow{\cdot \frac{1}{3}} \xrightarrow{\cdot \frac{1}{3}}$$

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example  $20, 10, \underline{5}, 5/2, 5/4, \dots$

recursive:  $a_1 = 20$

$$a_{n+1} = \frac{1}{2} a_n$$

explicit:

$$a_n = 20 \cdot \left(\frac{1}{2}\right)^{n-1}$$

$$\begin{array}{ccccccc} a_1 & a_2 & a_3 & a_4 & \dots & a_n \\ \xrightarrow{\cdot r} & \xrightarrow{\cdot r} & \xrightarrow{\cdot r} & \xrightarrow{\cdot r} & & \xrightarrow{\cdot r} \\ a_n = a_1 \cdot r^{n-1} \end{array}$$

check:  $a_3 = 20 \cdot \left(\frac{1}{2}\right)^2 = 5$

$$a_4 = 20 \cdot \left(\frac{1}{2}\right)^3 = 5/2$$

# Fibonacci sequence

1, 1, 2, 3, 5, 8, 13, 21, ...

recursive:

$$a_{n+1} = a_n + a_{n-1}$$

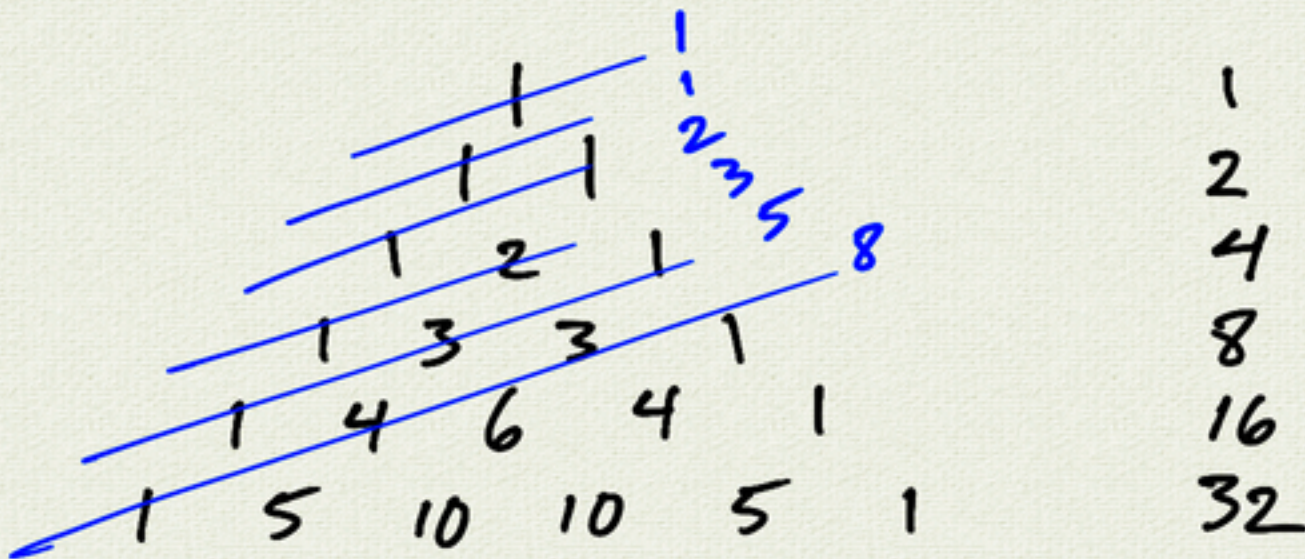
$$a_{n+2} = a_n + a_{n+1}$$

either

plus:

$$a_1 = 1$$

$$a_2 = 1$$



$$\binom{n}{0} + \binom{n}{1} + \dots + \binom{n}{k} + \dots + \binom{n}{n} = \underline{\underline{2^n}}$$

#subsets of size 0      #subsets of size 1      #subsets of size k      #subsets