53

$\boxed{\text{20 numbers picked}}$ | 80 cards

20 selected

win if: 3, 4, 5 or 20 match

$$P(\text{5 matches}) = \frac{\binom{20}{5}\binom{60}{15}}{\binom{80}{20}}$$

selected

15 not selected

hypergeometric

$$P(\geq 5 \text{ matches}) = P(5) + P(6) + P(7) + \ldots + P(20)$$

# 6.5 Sequences

$1, 2, 3, 4, 5, 6, \ldots$

$1, 3, 5, 7, \ldots$

$2, 4, 6, 8, \ldots$ $\Big] \to \infty$

$1, 1, 1, 1, \ldots \to 1$

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \ldots \to 0$

$0, 1, 0, 1, 0, 1, \ldots$ no limit

notation:

$a_1, a_2, a_3, a_4 \ldots, a_n$

nth term

$\{a_n\}$ $\{a_n\}_{n=1}^{\infty}$

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example:

$a_n = 2n$

$2, 4, 6, 8, 10, \ldots$

explicit formula

| $n$ | $a_n$ |
|---|---|
| 1 | 2 |
| 2 | 4 |
| 3 | 6 |

---

$$a_n = \left(\frac{1}{2}\right)^n$$

$\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}, \ldots$

$a_{n+1} = \frac{1}{2} \cdot a_n \Leftarrow$ recursive formula

# arithmetic sequences

$$a_1 \xrightarrow[+d]{} a_2 \xrightarrow[+d]{} a_3 \xrightarrow[+d]{} a_4$$

common difference

example:

$$3 \xrightarrow[+2]{}, 5 \xrightarrow[+2]{}, 7, 9, \ldots$$

common difference $= 2$

recursive formula:

$$a_{n+1} = (a_n) + 2 \quad , \quad a_1 = 3$$

explicit formula

$$\boxed{a_1 \xrightarrow[+d]{} a_2 \xrightarrow[+d]{} a_3 \xrightarrow[+d]{} a_4 \ldots \xrightarrow[+d]{} a_n}$$

$$a_n = a_1 + (n-1)d$$

$$3 \xrightarrow{+2} 5 \xrightarrow{+2} 7 \xrightarrow{+2} 9 \xrightarrow{+2} \boxed{11}$$

$$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5$$

$$\boxed{a_n = 3 + (n-1)2}$$

3, 5, 7, 9, ...

$$a_5 = \underbrace{a_1}_{3} + (\underbrace{5-1}_{4})\underbrace{2}_{d}$$

$$= 11$$

example:

$$a_{101} = 3 + (101-1) \cdot 2$$

$$= 203$$

example: arithmetic sequence

$a_3 = 6 \qquad a_8 = 21$

find the explicit formula (and the first few terms)

$$\frac{\quad}{a_1} \; \frac{\quad}{a_2} \; \frac{6}{a_3} \; \frac{\quad}{a_4} \; \frac{\quad}{a_5} \; \frac{\quad}{a_6} \; \frac{\quad}{a_7} \; \frac{21}{a_8}$$

$$a_n = a_1 + (n-1)d$$

$$6 = a_3 = a_1 + 2d$$
$$21 = a_8 = a_1 + 7d$$

$$15 = 5d$$
$$\Rightarrow d = 3$$

$$6 = a_1 + 2 \cdot 3$$
$$\Rightarrow a_1 = 0$$

$$a_n = 0 + (n-1)3$$
$$\boxed{a_n = 3n - 3}$$ explicit formula

$0, 3, 6, 9, \ldots$

# geometric sequence

$a_1 \xrightarrow{\cdot r} a_2 \xrightarrow{\cdot r} a_3 \xrightarrow{\cdot r} a_4$

common ratio

**example:**

$1 \xrightarrow{\cdot 2} 2 \xrightarrow{\cdot 2} 4 \xrightarrow{\cdot 2} 8, 16, 32,$

$1 \xrightarrow{1/3} \frac{1}{3} \xrightarrow{1/3} \frac{1}{9} \xrightarrow{1/3} \frac{1}{27}, \frac{1}{81}, \ldots$

---

**example** $20, 10, \underline{5}, 5/2, 5/4, \ldots$

recursive: $a_1 = 20$

$a_{n+1} = \frac{1}{2} a_n$

$a_1 \xrightarrow{\cdot r} a_2 \xrightarrow{\cdot r} a_3 \xrightarrow{\cdot r} a_4 \ldots \xrightarrow{\cdot r} a_n$

$a_n = a_1 \cdot r^{n-1}$

explicit:

$$a_n = 20 \left(\frac{1}{2}\right)^{n-1}$$

check: $a_3 = 20 \cdot \left(\frac{1}{2}\right)^2 = 5$

$a_4 = 20 \cdot \left(\frac{1}{2}\right)^3 = 5/2$

# Fibonacci sequence

$$1, 1, 2, 3, \underline{5}, \underline{8}, 13, 21, \ldots$$

recursive:

$$a_{n+1} = a_n + a_{n-1}$$
$$a_{n+2} = a_n + a_{n+1}$$

either

plus:
$a_1 = 1$
$a_2 = 1$

---

1, 1, 2, 3, 5, 8

| | |
|---|---|
| 1 | 1 |
| 1 1 | 2 |
| 1 2 1 | 4 |
| 1 3 3 1 | 8 |
| 1 4 6 4 1 | 16 |
| 1 5 10 10 5 1 | 32 |

$$\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{k} + \cdots + \binom{n}{n} = \underline{\underline{2^n}}$$

#subsets of size 0

#subsets of size 1

#subsets of size k

#subsets