11.3
#17 geometric sequence

$$\underline{a_1} \xrightarrow{\times r} a_2 \xrightarrow{\times r} a_3$$

common ratio

$a_6 = 25$

$a_8 = 6.25$

$a_n = a_1 \cdot r^{n-1}$

$$\left.\begin{aligned} 25 = a_6 = a_1 \cdot r^5 \\ 6.25 = a_8 = a_1 r^7 \end{aligned}\right] \frac{25}{6.25} = \frac{1}{r^2}$$

eliminate

$$r^2 = \frac{6.25}{25} = \frac{1}{4}$$

$$\Rightarrow r = \frac{1}{2}$$

$$\begin{tikzcd} \underset{a_6}{\underline{25}} \arrow[r, "r"'] & \underset{a_7}{\underline{12.5}} \arrow[r, "r"'] & \underset{a_8}{\underline{6.25}} \end{tikzcd}$$

$25 = a_6 = a_1 \cdot \left(\frac{1}{2}\right)^5$

find a_1

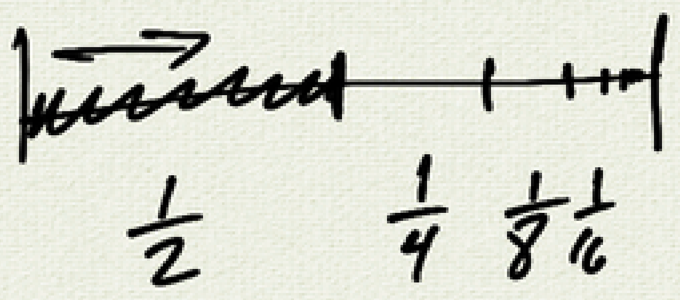
$25 = a_1 \cdot \frac{1}{32}$

$a_1 = 25 \cdot 32 = 800 \Rightarrow a_n = 800\left(\frac{1}{2}\right)^{n-1}$

a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	
800	400	200	100	50	25	12.5	6.25	...

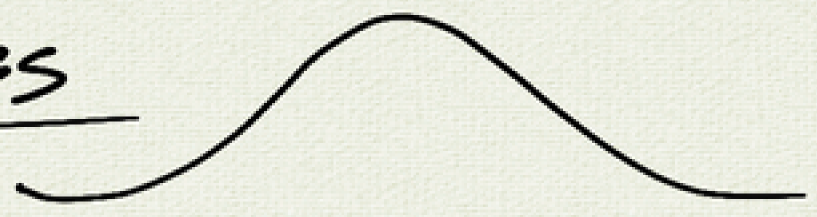
6.6 Series

Zeno's Paradox



$$\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\dots \longrightarrow 1$$

Gauss



$$\underbrace{1+\underbrace{2+\underbrace{3+\dots+98}_{101}+99}_{101}+100}_{101}=5050$$

$$\underset{\text{pairs}}{50}\cdot 101$$

$$\left[=\left(\frac{1+100}{2}\right)\cdot 100\right]$$

$$1\xrightarrow{+1}2\xrightarrow{+1}3,4,5,6,\dots$$ arithmetic

arithmetic:

$$S_n=a_1\xrightarrow{+d}a_2\xrightarrow{+d}a_3\rightarrow a_4\dots a_{n-1}\xrightarrow{+d}a_n$$

Sum of the first n terms

$$\boxed{S_n=\frac{(a_1+a_n)}{2}\cdot n}$$

example: 3, 5, 7, 9, 11, 13, ...

find $S_5=a_1+a_2+\dots+a_5$

$=3+5+7+9+11$ (I could do this)

$\left(\text{or } S_5=\frac{(3+11)}{2}\cdot 5=35\right)$

$\boxed{a_{100}=3+99\cdot 2 \\ =201}$

$$S_{100}=a_1+a_2+a_3+\dots+a_{99}+a_{100}$$

$=3+\dots \quad +201$

$=\frac{(3+201)}{2}\cdot 100$

$=10200$

$3+5=8$

$4+4$

$n=2$

$\underbrace{3+\underbrace{5+7}+9}$

$\underbrace{6+\underbrace{6+6}+6}$

$n=4$

notation:

$$S_{100}=a_1+a_2+\dots+a_{99}+a_{100}$$

$$=\sum_{k=1}^{100}a_k$$

summation notation

Σ sigma $\Leftrightarrow$ Sum

$$a_k=3+2(k-1)$$

$$\Rightarrow S_{100}=\sum_{k=1}^{100}a_k$$

$$=\sum_{k=1}^{100}3+2(k-1)$$

$$S_5=a_1+a_2+a_3+a_4+a_5$$

$$=\sum_{k=1}^{5}a_k$$

$$=\sum_{k=1}^{5}3+2(k-1)$$

$a_n=a_1+d(n-1)$

$a_k=a_1+d(k-1)$

$$=\boxed{\sum_{k=1}^{5}1+2k}$$

k ends at 5

k starts at 1

$3+5+7+9+11$

$k=1$ $k=2$ $k=5$

polynomial multiplication

$$(1-x)(1+x+x^2+\dots+x^{n-1})$$

$$= 1+x+x^2+\dots+x^{n-1}$$

$$-x-x^2-\dots-x^{n-1}-x^n$$

$$= 1-x^n$$

$$\Rightarrow 1+x+x^2+\dots+x^{n-1} = \frac{1-x^n}{1-x}$$

geometric sequence

$$a_1 \xrightarrow[\times r]{} a_2 \xrightarrow[\times r]{} a_3 \quad a_4 \quad \dots$$

$$a_1 \quad a_1r \quad a_1r^2 \quad a_1r^3 \quad \dots$$

$$S_n = \sum_{k=1}^{n} a_k$$

$$= a_1+a_2+\dots+a_n$$

$$= a_1+a_1r+a_1r^2+\dots+a_1r^{n-1}$$ (sum of 1st n terms)

$$= a_1(1+r+r^2+\dots+r^{n-1})$$

$$\boxed{S_n = a_1\frac{(1-r^n)}{1-r}}$$

finite sum of geometric sequence

example: $\frac{1}{2} \xrightarrow[r=1/2]{} \frac{1}{4} \xrightarrow{} \frac{1}{8}, \frac{1}{16}, \dots$

$$a_n = \frac{1}{2}\left(\frac{1}{2}\right)^{n-1} = \left(\frac{1}{2}\right)^n$$

$$S_{10} = a_1\frac{(1-r^n)}{1-r}$$

$$= \frac{\boxed{\frac{1}{2}}\left(1-\left(\frac{1}{2}\right)^{10}\right)}{\underbrace{\boxed{(1-1/2)}}_{1/2}}$$

$$= 1-\frac{1}{1024}$$

$$= \frac{1023}{1024}$$

$2^{10}=1024$

binary thousand

$$\boxed{(1-x)}\boxed{(1+x+x^2+x^3+\dots)}$$

$$= 1+x+x^2+x^3+\dots$$

$$\quad -x-x^2-x^3-\dots$$

$$= 1$$

$$\Rightarrow \boxed{1+x+x^2+\dots} = \frac{1}{\boxed{1-x}}$$

geometric:

$$S_\infty = a_1 + a_1 r + a_1 r^2 + \dots$$

$$= a_1(1+r+r^2+r^3+\dots)$$

$$\boxed{S_\infty = \frac{a_1}{1-r}} \Leftarrow$$ valid when $|r|<1$

example: $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\dots = S_\infty$

$$a_1 = \frac{1}{2} \qquad r = \frac{1}{2}$$

$$S_\infty = \frac{a_1}{1-r} = \frac{1/2}{1-1/2} = 1$$

$$\lim_{n\to\infty}\left(\frac{1}{2}\right)^n = 0$$

$$\boxed{S_n = \frac{a_1(1-r^n)}{1-r}} \qquad (n\to\infty)$$

$$= \left(1 - \underline{\underline{\left(\frac{1}{2}\right)^n}}\right)$$

$$S_\infty = \lim_{n\to\infty} S_n = 1$$

$\lim_{n\to\infty} S_n$ exists $\iff |r|<1$

$$S_\infty = 1 + 2 + 4 + 8 + \dots$$

$$\xrightarrow[r=2]{} \quad \xrightarrow[r=2]{}$$

no finite limit

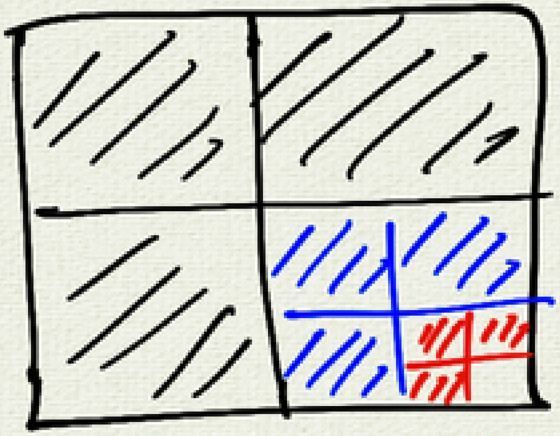
$|r|>1$

$$1+\frac{1}{3}+\frac{1}{27}+\frac{1}{81}+\dots$$

$$\xrightarrow[1/3]{} \quad \xrightarrow[1/3]{}$$

finite limit

$|r|<1$



$$\underset{\substack{\nearrow \\ a_1 = \frac{3}{4}}}{\frac{3}{4}} + \left(\frac{3}{4}\right)\underset{\substack{\nwarrow \\ r = \frac{1}{4}}}{\frac{1}{4}} + \frac{3}{4}\left(\frac{1}{4}\right)\left(\frac{1}{4}\right) + \dots$$

$$S_\infty = \frac{a_1}{1-r}$$

$$= \frac{3/4}{1 - 1/4}$$

$$= 1$$

summation notation:

$$S_\infty = \sum_{k=1}^{\infty} \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^{k-1}$$

$$= \frac{3}{4} + \frac{3}{4}\left(\frac{1}{4}\right) + \left(\frac{3}{4}\right)\left(\frac{1}{4}\right)^2 + \dots$$