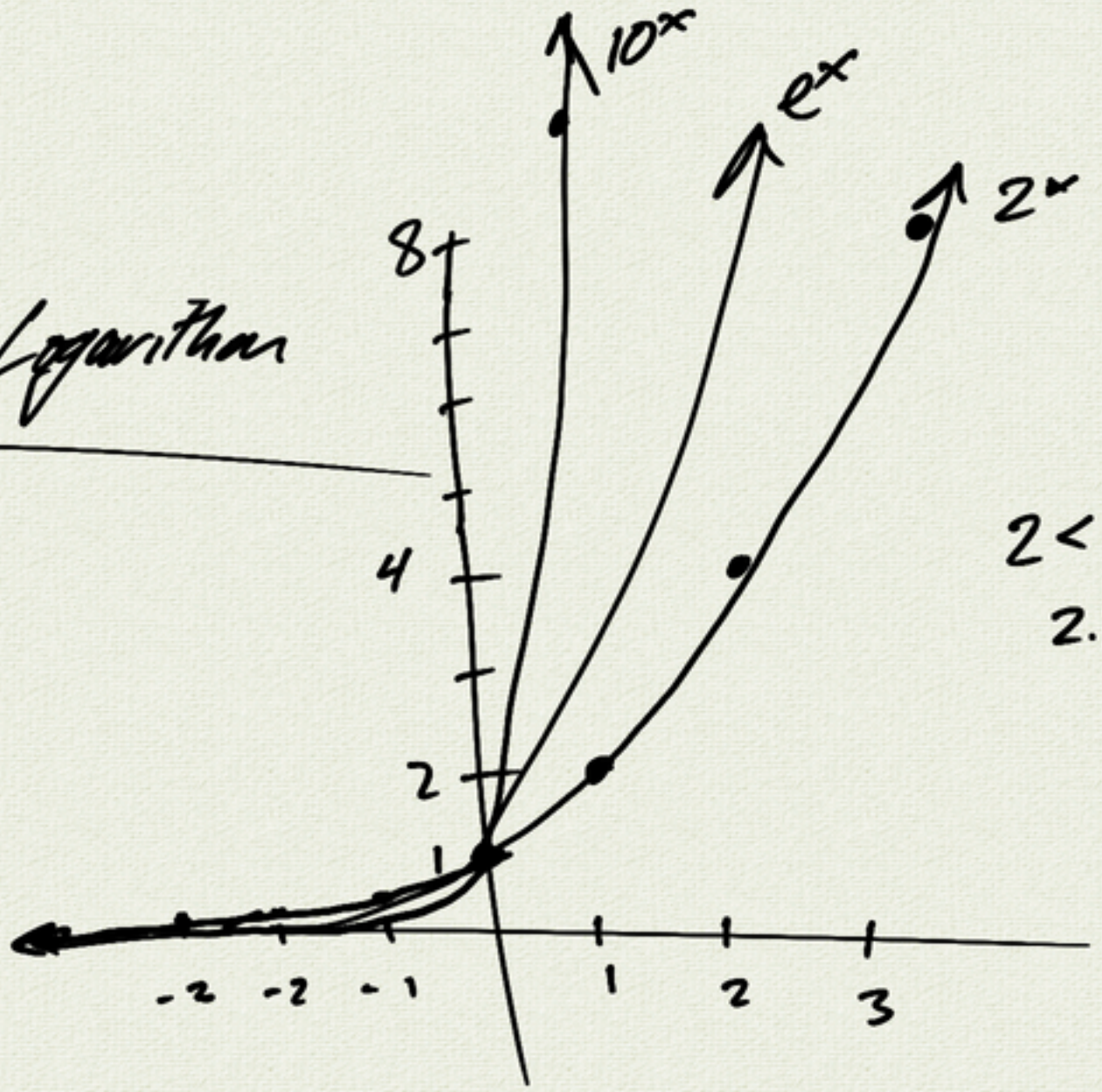
7.1 Exponential & Logarithm

x	2^x
0	1
1	2
2	4
3	8
4	16
-1	1/2
-2	1/4
-3	1/8



$2 < e < 10$

$2.718\ldots$

$\lim_{x \to \infty} 2^x = \infty$

$\lim_{x \to -\infty} 2^x = 0$

---

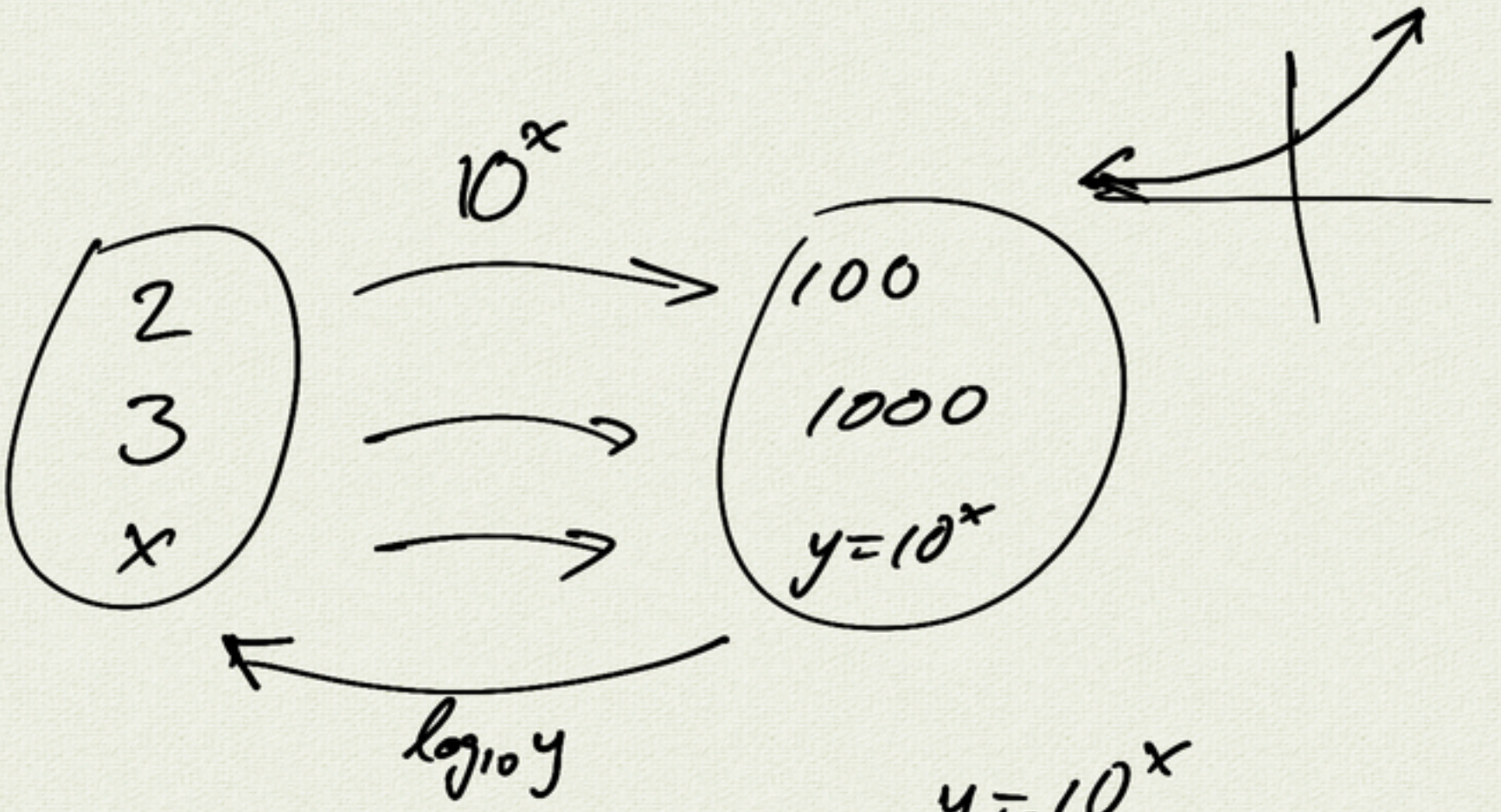
properties: $2^x 2^y = 2^{x+y}$

$2^5 \cdot 2^3$ $\quad \underbrace{\underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}_{2^5} \underbrace{2 \cdot 2 \cdot 2}_{2^3}}_{2^8}$

$(2^x)^y = 2^{xy}$

$(2^3)^2 = (2 \cdot 2 \cdot 2)^2$

$= \underbrace{(2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2)}_{2^6}$



$\log_{10} 100 = 2$

$\log_{10} 1000 = 3$

$y = 10^x$

$\Updownarrow$

$x = \log_{10} y$

$2 \longleftrightarrow 100 = 10^2$

$3 \longleftrightarrow 1000 = 10^3$

$2+3=5 \longleftrightarrow (10^2)(10^3)$ $\qquad 10^2 10^3 = 10^{2+3}$

add mult

$= 10^5$

---

log properties:

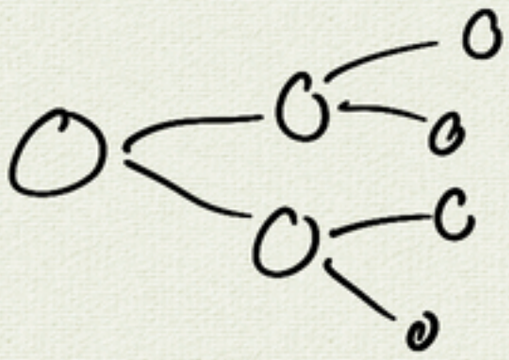
$$\log(xy) = \log x + \log y$$

$$\log(x^y) = y \log(x)$$

bacterial growth

starting population $P_0 = 1000$ cells

doubling time = 3 hours



model/equation:

$$P(t) = 1000 \cdot 2^{(t/3)}$$

population

t	$P(t)$	
0	1000	×2
3	2000	×2
6	4000	
9	8000	

$t/3$: # of doubling times

$$P(15) = 1000 \cdot 2^{15/3}$$
$$= 1000 \cdot 2^5$$
$$= 32000$$

$$P(t) = P_0 e^{kt} \quad \Leftarrow \text{base } e$$

Same problem: $P_0 = 1000$

doubling time 3

$\Rightarrow$ find k

$$P(t) = 1000 e^{kt}$$

$$1000 = 1000 e^0 \quad \Leftarrow \text{no help}$$

$$2000 = 1000 e^{k \cdot 3}$$

$$2 = e^{3k}$$

ln both sides

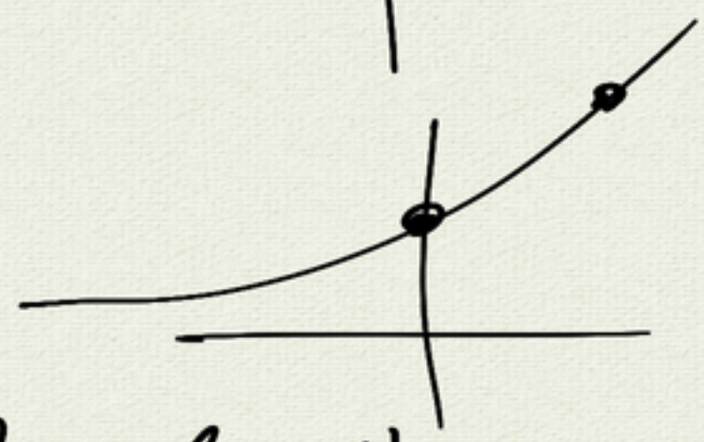
$$\ln 2 = 3k$$

$$k = \frac{\ln 2}{3}$$

$$\Rightarrow \boxed{P(t) = 1000 e^{\frac{\ln 2}{3} t}}$$

$$P(15) = 1000 e^{\frac{\ln 2}{3} \cdot 15}$$
$$= 1000 e^{5 \ln 2}$$
$$= 1000 e^{\ln 32}$$
$$= 32000$$

t	$P(t)$
0	1000
3	2000



$\ln x = \log_e x$

math: $\log = \log_{10}$

code: $\log = \log_e$

$\log_{10} 10^x = x$

$10^{\log_{10} x} = x$) inverse functions

$\ln e^x = x$

$e^{\ln x} = x$) inverse function

$\ln 2^5 = 5 \ln 2$

for exponential functions, we can use any base

Logarithm change of base

$$\log_b y = x \iff b^x = y$$

some base b (note, not 10)

$$\ln b^x = \ln y$$

$$x \ln b = \ln y$$

$$x = \frac{\ln y}{\ln b}$$

$$\log_b y = \frac{\ln y}{\ln b} \quad \left(= \frac{\log_{10} y}{\log_{10} b} \right)$$

convert any base to any other base