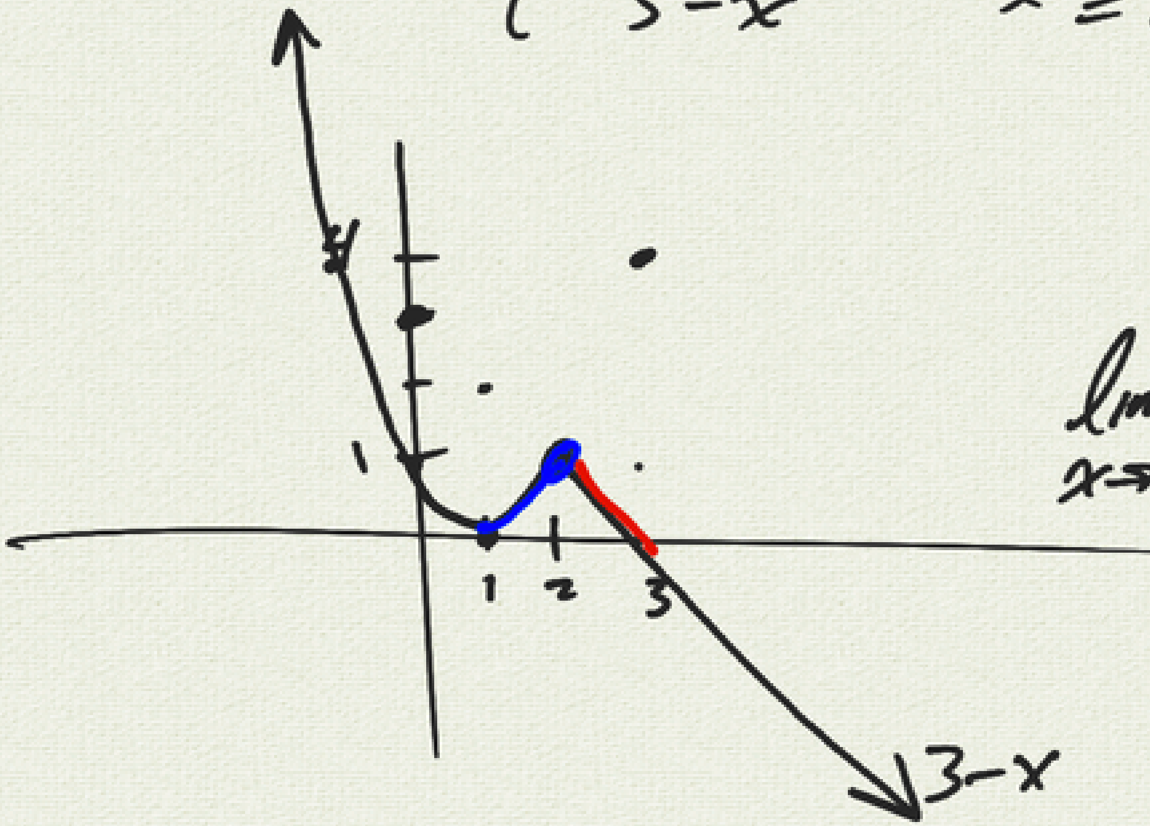
2.3

117

$$h(x) = \begin{cases} x^2 - 2x + 1 & x < 2 \\ 3 - x & x \geq 2 \end{cases}$$



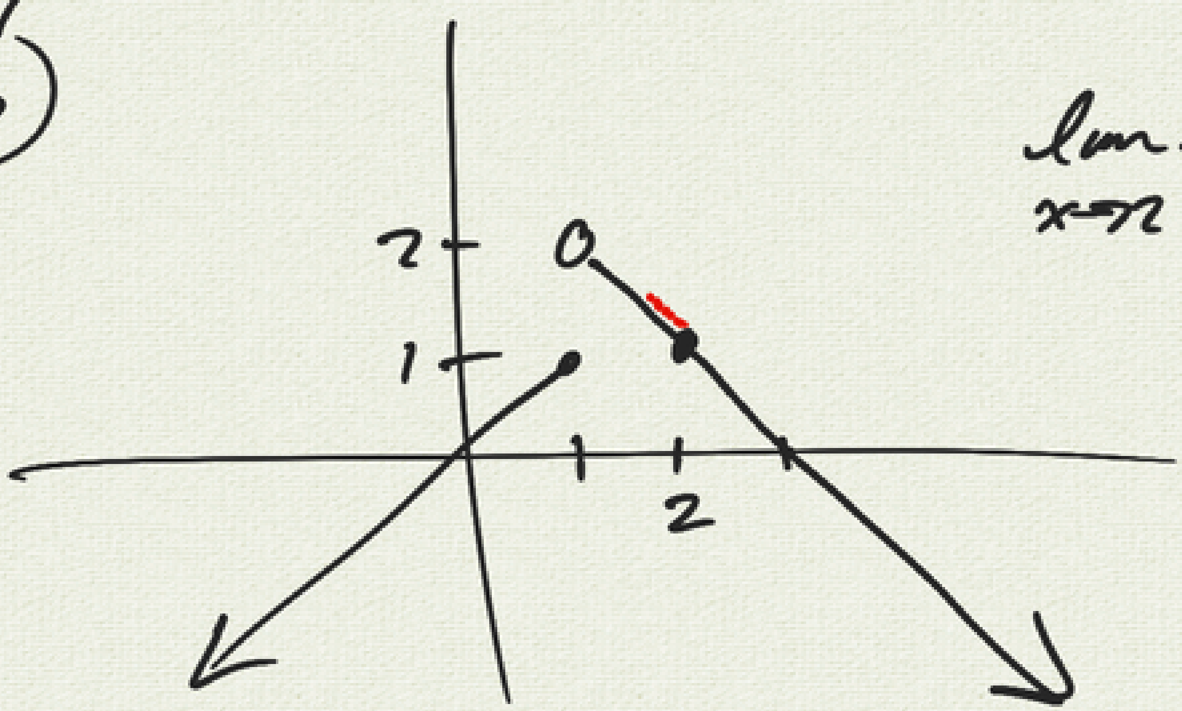
$x^2 - 2x + 1 = (x-1)^2$

$\lim_{x \to 2^-} h(x) = 1$

$\lim_{x \to 2^+} h(x) = 1$

2.2

53



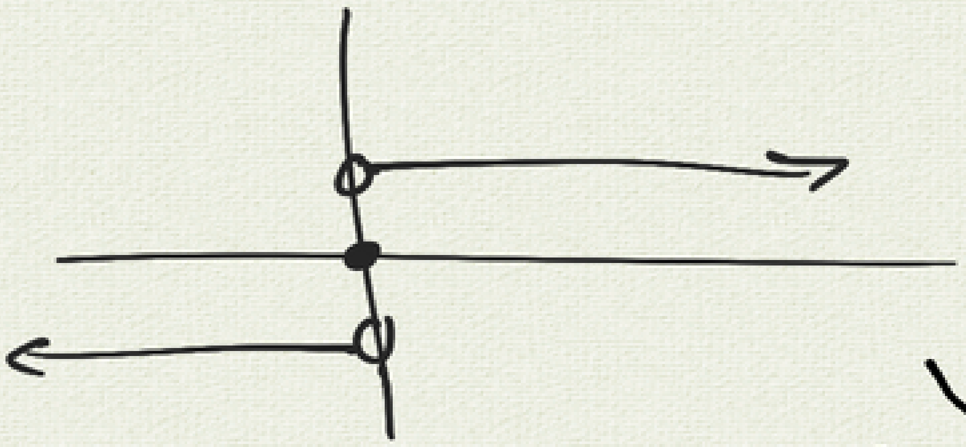
$\lim_{x \to 2} f(x) = 1$

8.2 Continuity

discontinuities

jump

$$f(x)=\begin{cases}\dfrac{x}{|x|} & \text{if } x\neq 0\\ 0 & \text{if } x=0\end{cases}$$

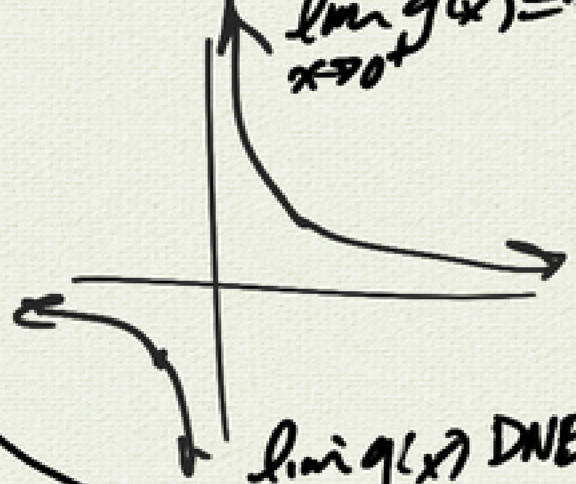


$\lim_{x\to 0^-} f(x) = -1 \neq 1 = \lim_{x\to 0^+} f(x)$

$\lim_{x\to 0} f(x)$ does not exist $\Rightarrow$ discontinuous at $x=0$

infinite

$$g(x)=\frac{1}{x}$$

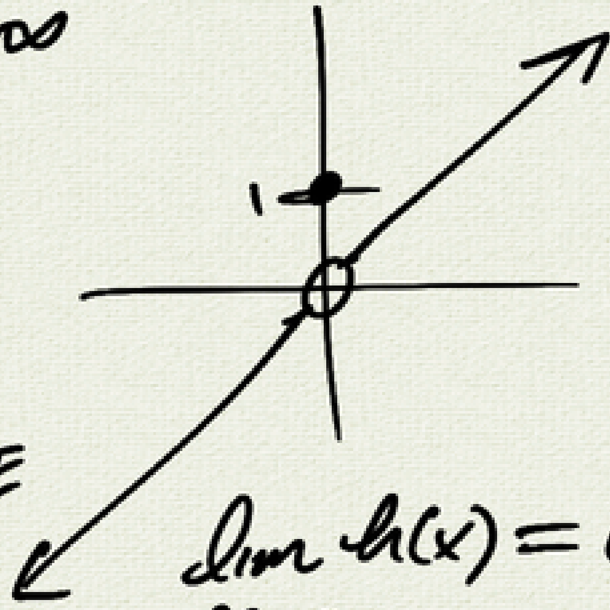


$\lim_{x\to 0^+} g(x)=\infty$

$\lim_{x\to 0} g(x)$ DNE

removable

$$h(x)=\begin{cases} x & \text{if } x\neq 0\\ 1 & \text{if } x=0\end{cases}$$



$\lim_{x\to 0} h(x) = 0 \neq h(0)$

h is discontinuous at $x=0$

$\lim_{x\to 0^-} h(x) = \lim_{x\to 0^+} h(x)$

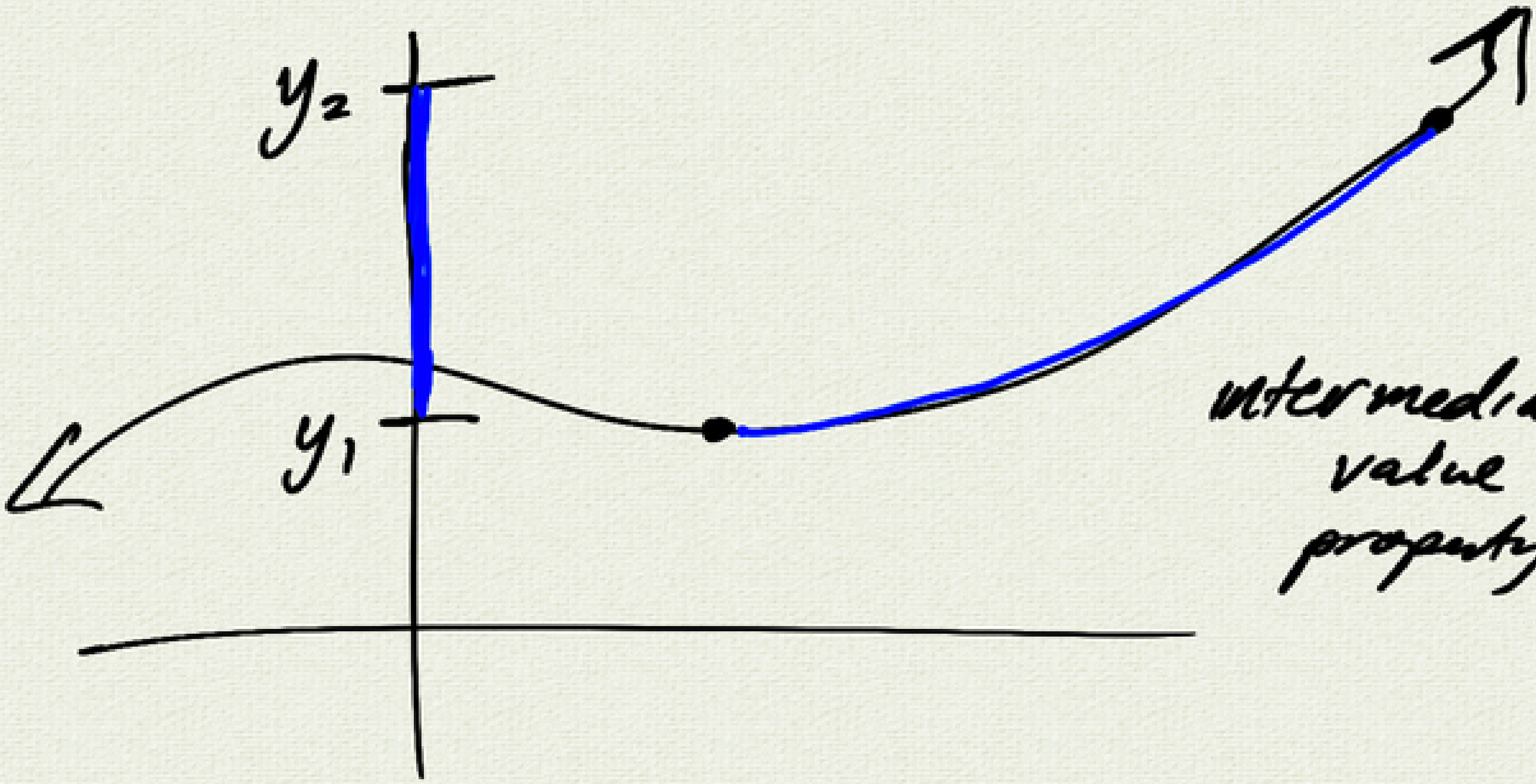
$\Rightarrow \lim_{x\to 0} h(x)$ exists

def: f is continuous at $x=a$

if $\lim_{x\to a} f(x) = f(a)$

(exists) (exists)

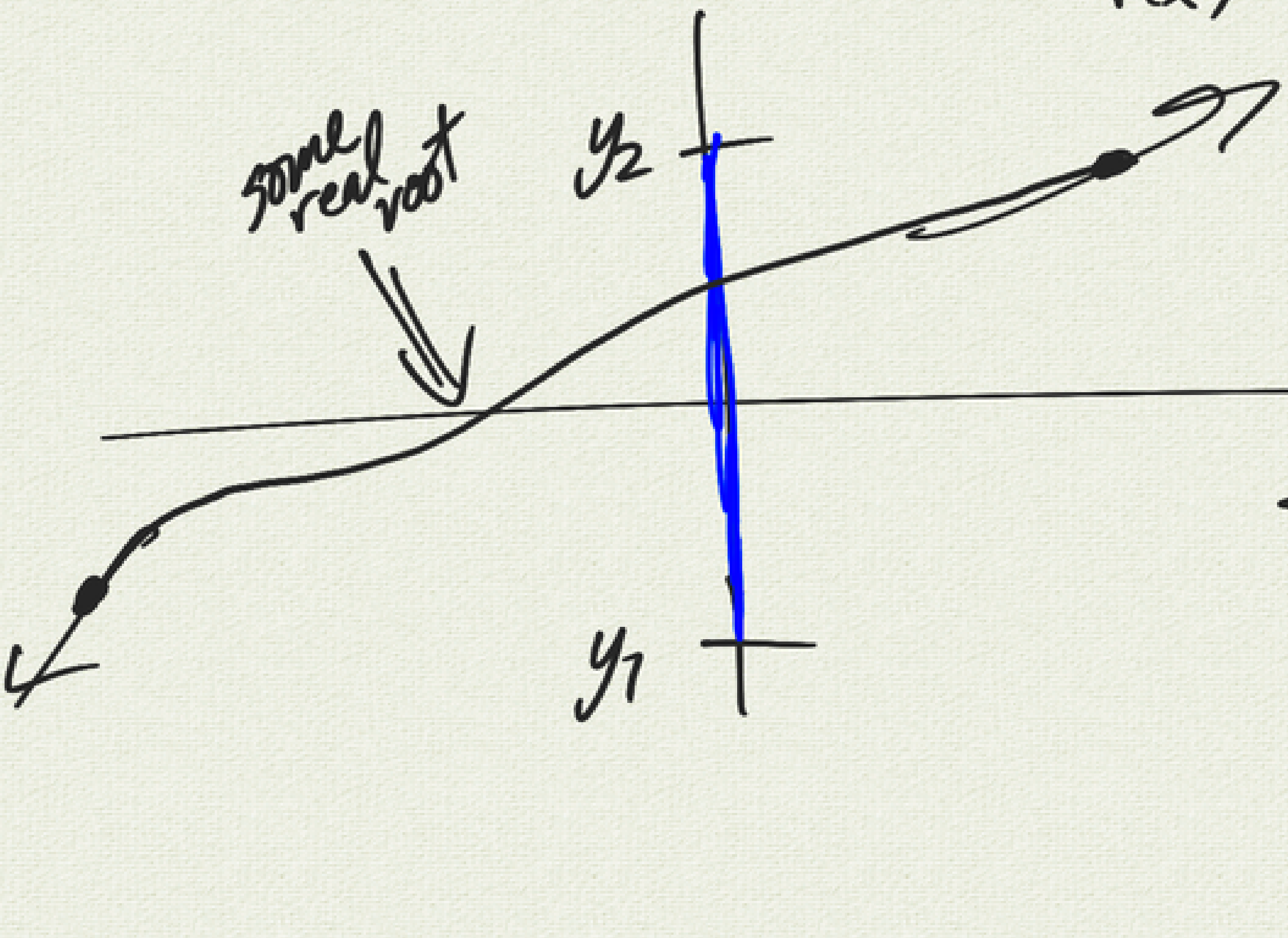
$\lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x)$

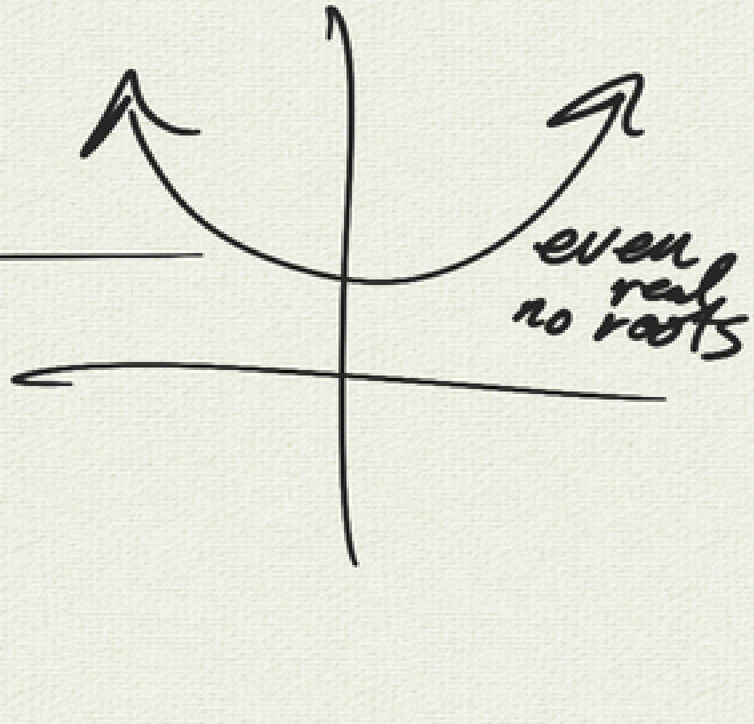


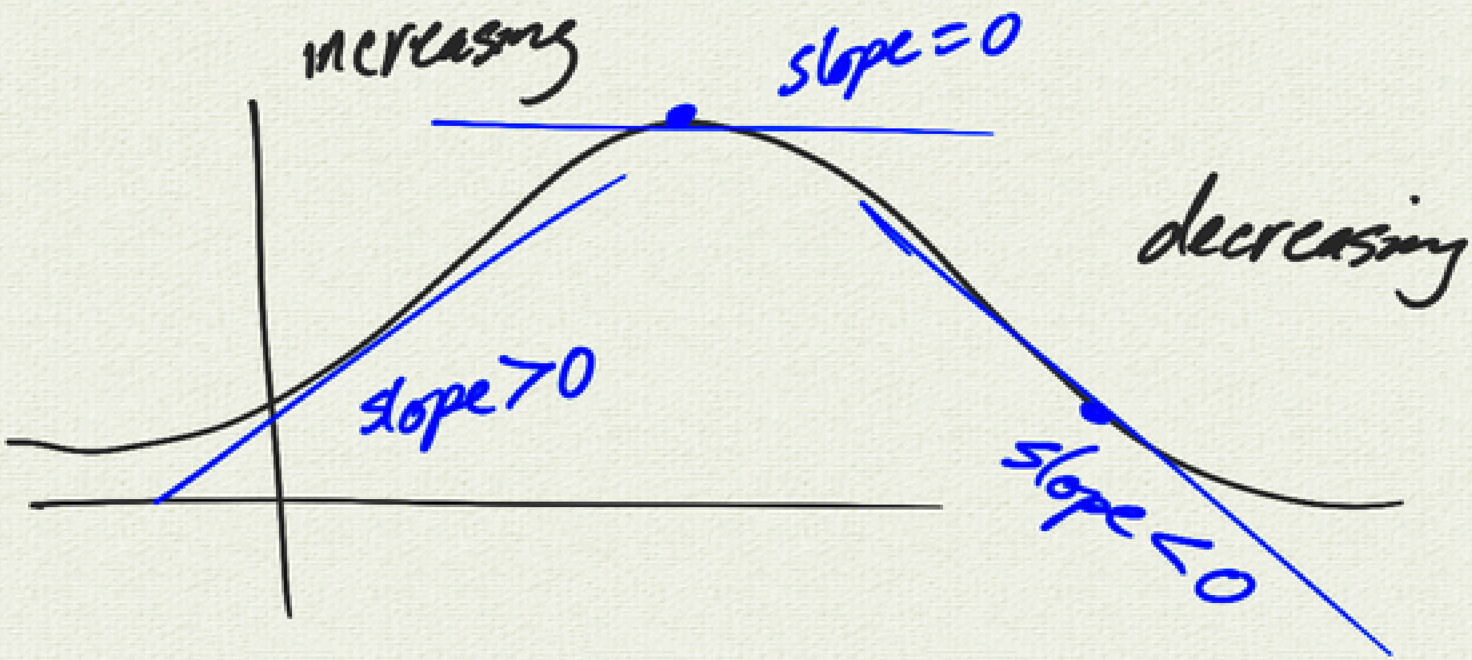
intermediate
value
property (theorem)

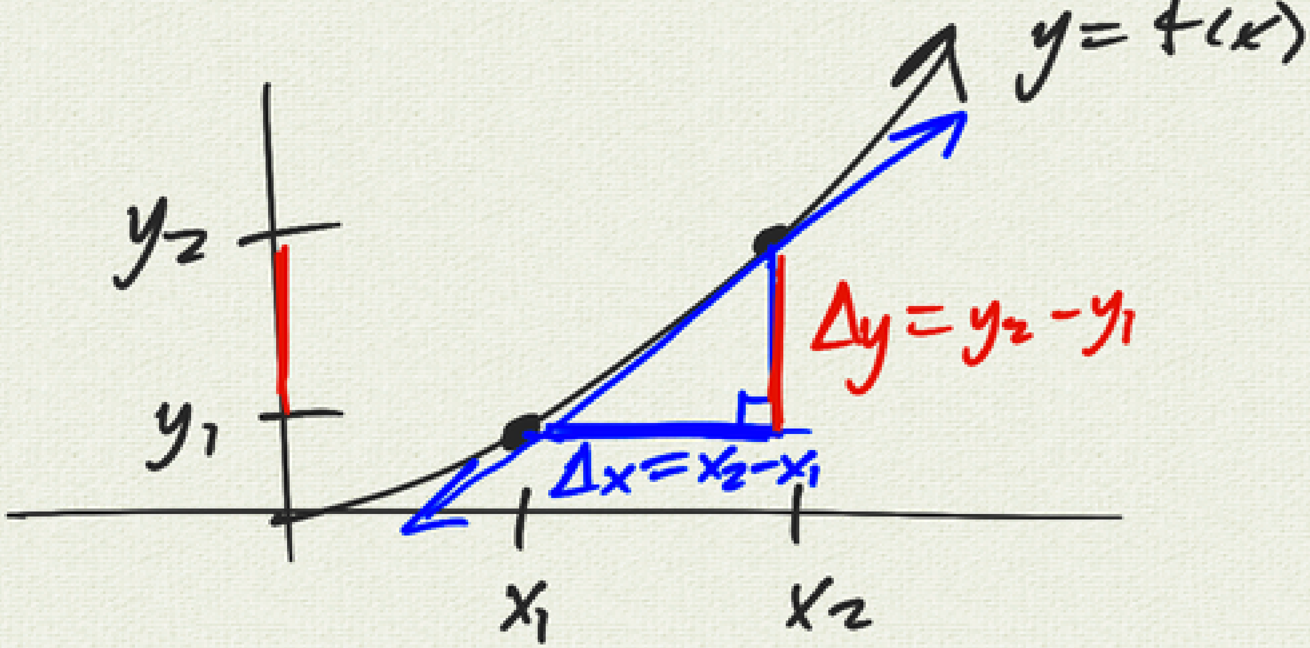
<u>Observation</u>: if $f(x)$ polynomial, and $\deg(f)$ is odd, then f has a real root

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots$$









slope of secant line

$$= \frac{\Delta y}{\Delta x}$$

$$= \frac{y_2 - y_1}{x_2 - x_1}$$