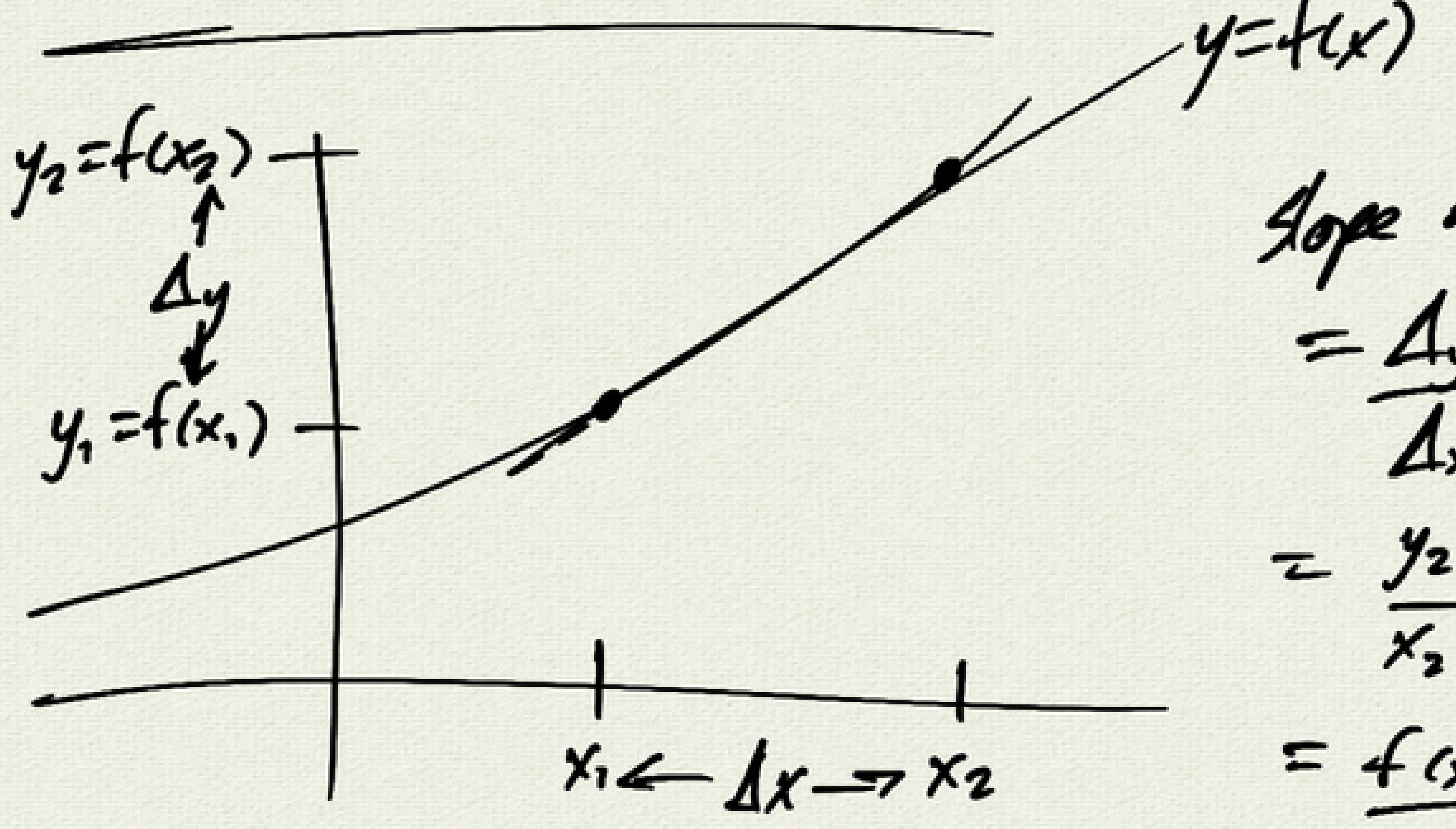
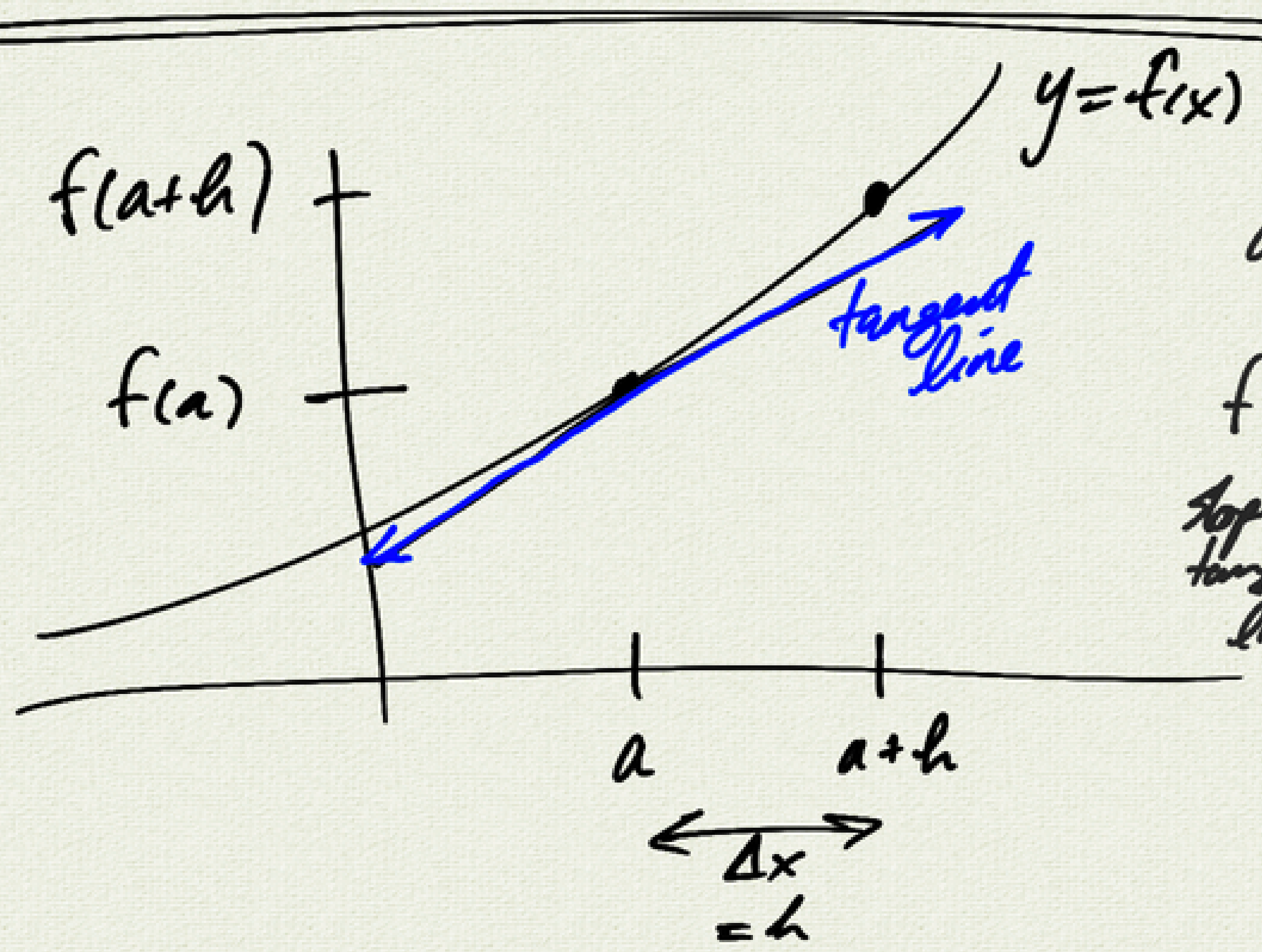
8.3 The Derivative



slope of secant line

$$= \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$



define:

$$f'(a) = \lim_{h \to 0} \boxed{\frac{f(a+h) - f(a)}{h}}$$

slope of tangent line

slope of secant line

example

$\boxed{f(x) = x^2}$ $\quad f(2+h) = (2+h)^2$

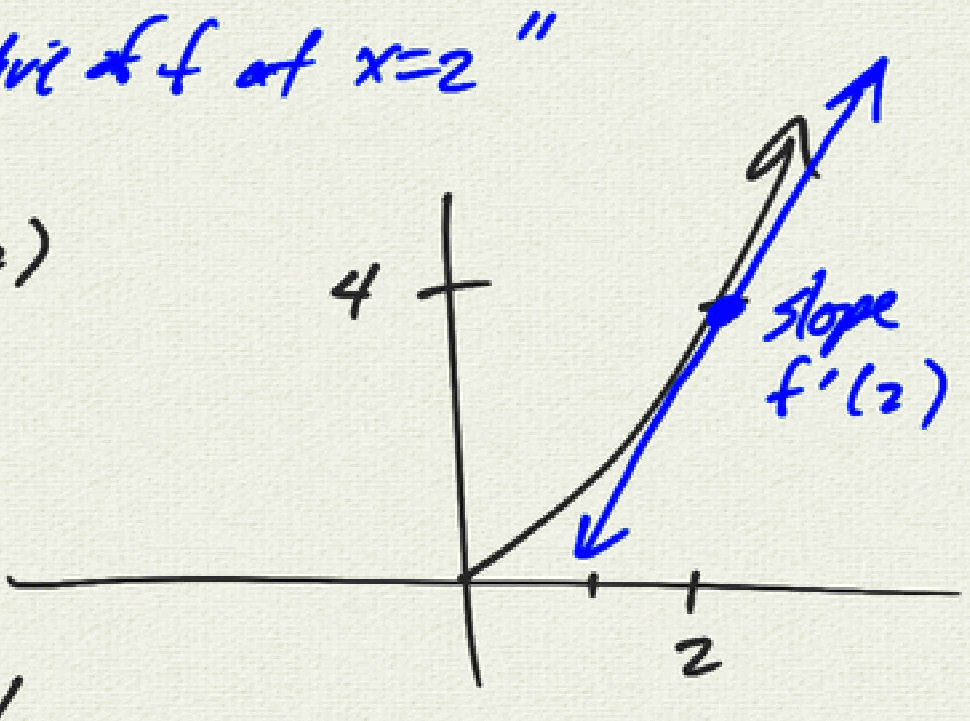
find slope of tangent line at $x=2$

$a=2$

$f'(a)$

$f'(2)$ "the derivative of f at $x=2$"

$$\begin{aligned}
f'(2) &= \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} \\
&= \lim_{h \to 0} \frac{(2+h)^2 - 2^2}{h} \\
&= \lim_{h \to 0} \frac{4 + 4h + h^2 - 4}{h} \\
&= \lim_{h \to 0} \frac{4h + h^2}{h} \\
&= \lim_{h \to 0} \frac{h(4+h)}{h} \\
&= \lim_{h \to 0} 4+h \\
&= 4
\end{aligned}$$

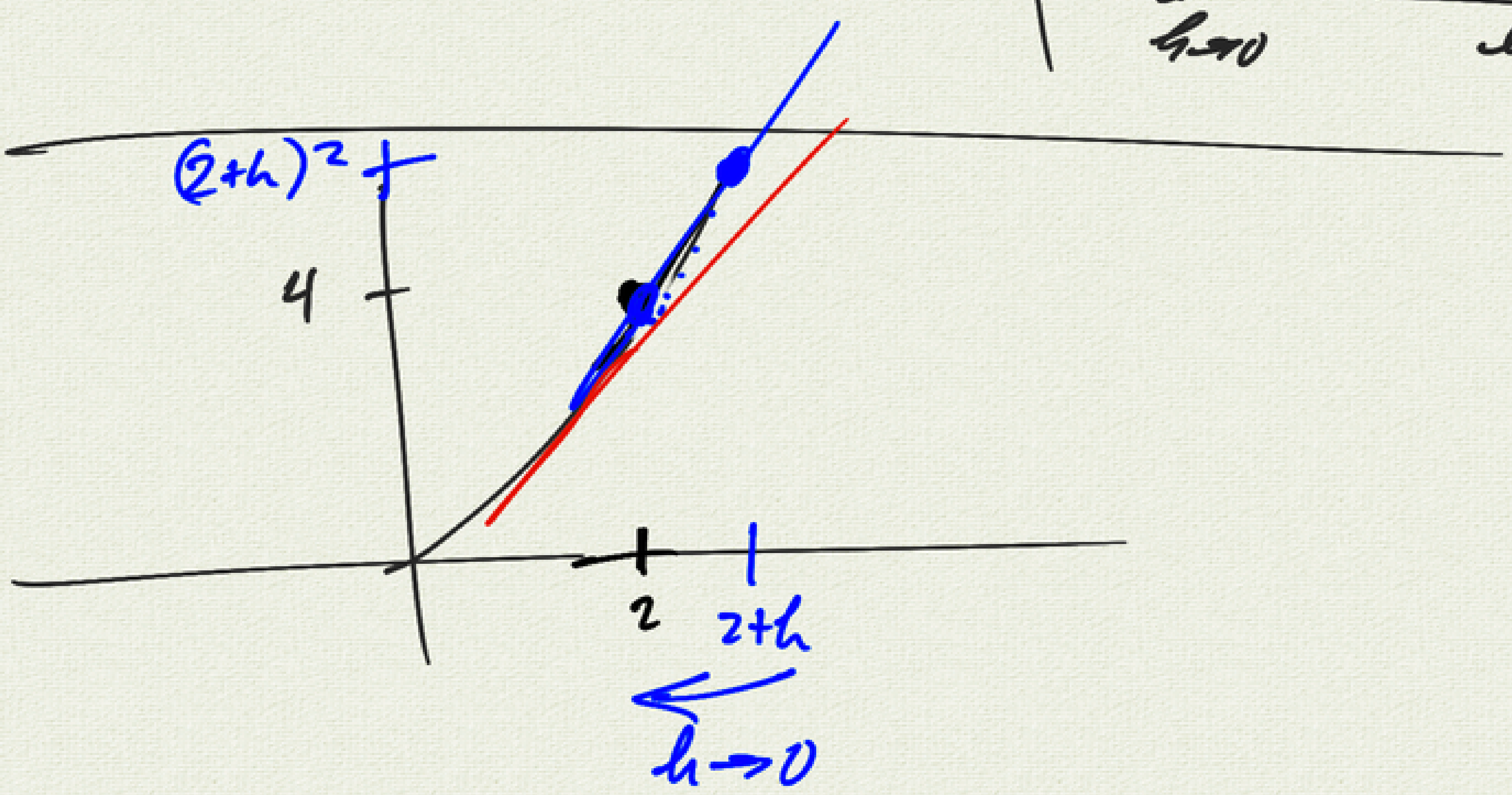


slope of secant line:

$$\frac{f(a+h) - f(a)}{h}$$

slope of tangent line:

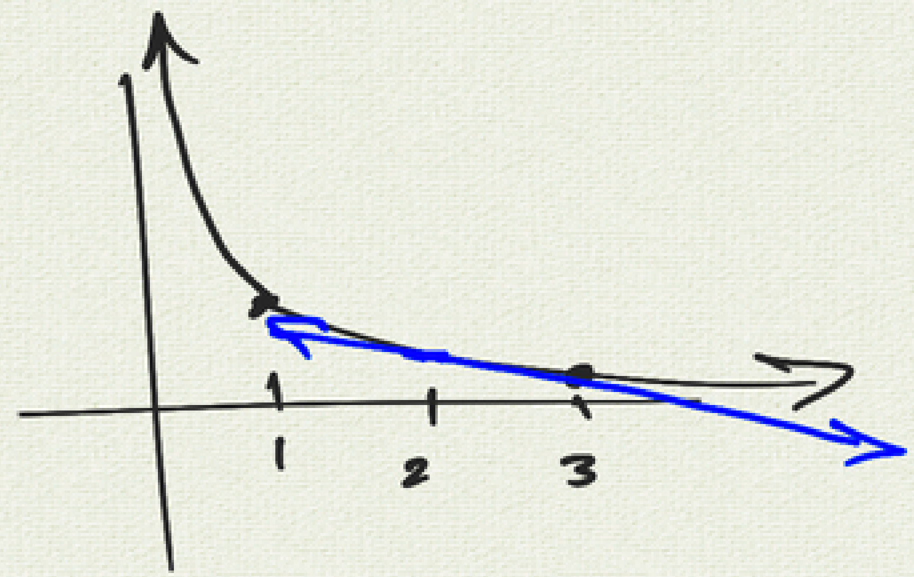
$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$



example 2

$$g(x) = \frac{1}{x}$$

find $g'(3)$ ⟸ Slope of tangent line at $x=3$



$$g'(3) = \lim_{h\to 0} \frac{g(3+h) - g(3)}{h}$$

$$= \lim_{h\to 0} \left[\frac{1}{3+h} - \frac{1}{3}\right]\frac{1}{h}$$

$$= \lim_{h\to 0} \frac{3-(3+h)}{(3+h)3}\frac{1}{h}$$

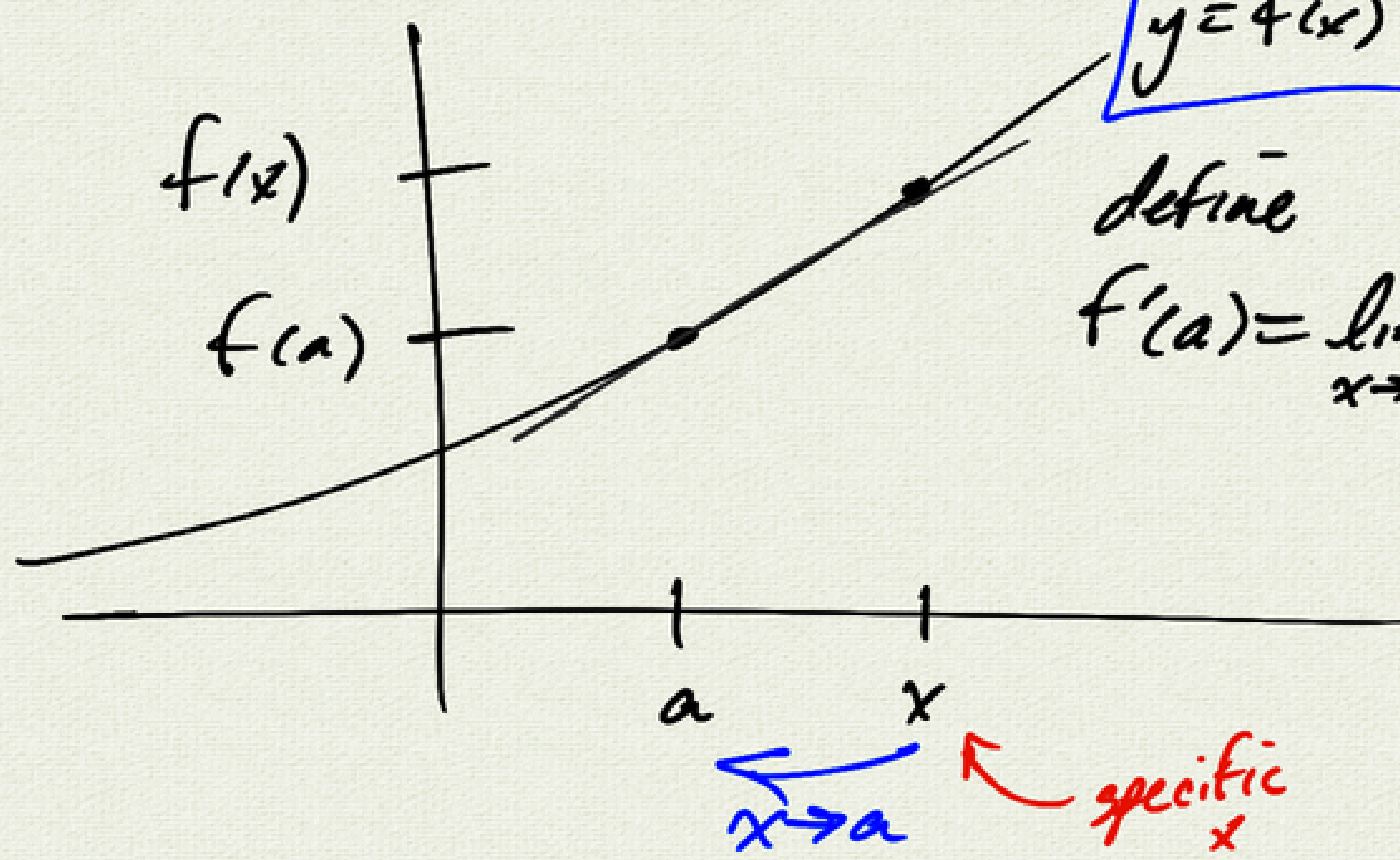
$$= \lim_{h\to 0} \frac{-h}{(3+h)3\,h}$$

$$= \lim_{h\to 0} -\frac{1}{(3+h)3}$$

$$\boxed{g'(3) = -\frac{1}{9}}$$

alternate definition (of the derivative)

$\boxed{y=f(x)}$ general x



define

$$f'(a)=\lim_{x\to a}\frac{f(x)-f(a)}{x-a}$$

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example 2 again:

$$g(x)=\frac{1}{x}$$

find $g'(3)$ (use alternate def.)

$$g'(3)=\lim_{x\to a}\frac{g(x)-g(a)}{x-a} \leftarrow a=3$$

$$=\lim_{x\to a}\frac{\frac{1}{x}-\frac{1}{a}}{x-a}$$

$$=\lim_{x\to a}\left(\frac{1}{x-a}\right)\left[\frac{1}{x}-\frac{1}{a}\right]$$

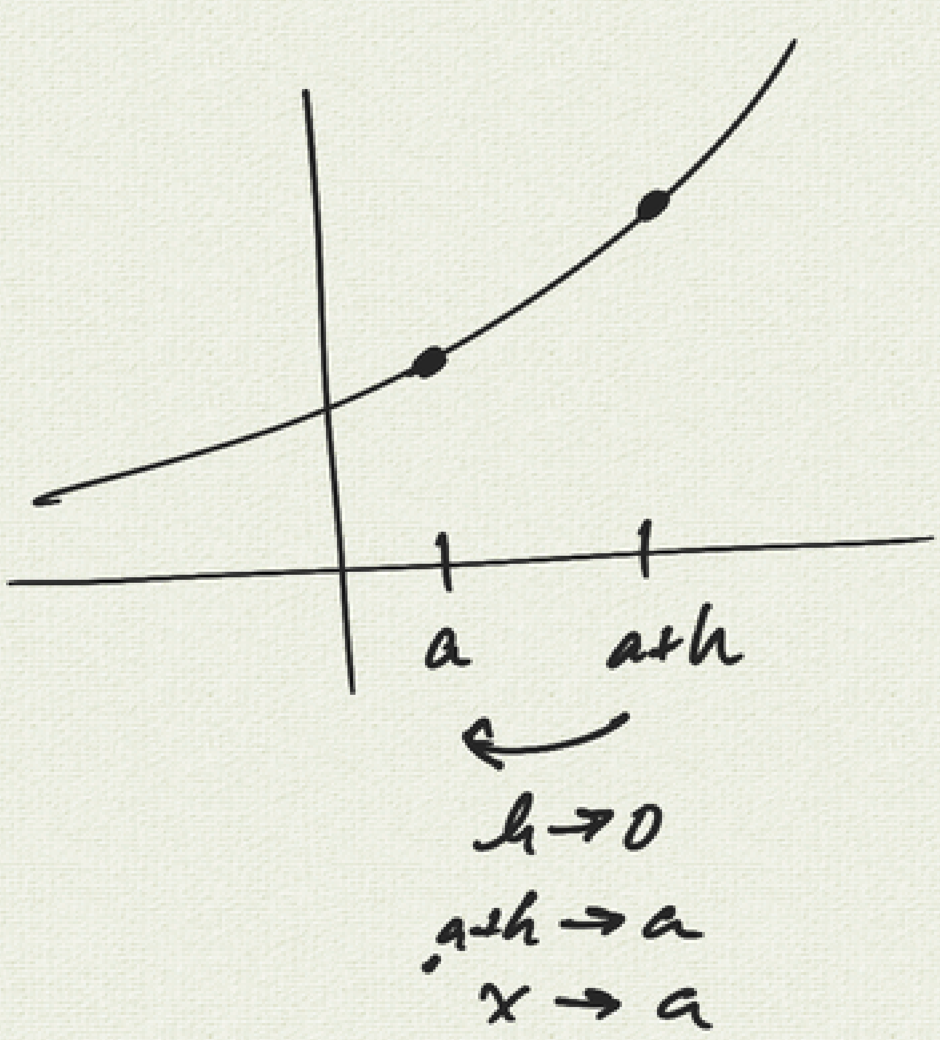
$$=\lim_{x\to a}\frac{1}{x-a}\,\frac{a-x}{ax} \qquad a-x=-(x-a)$$

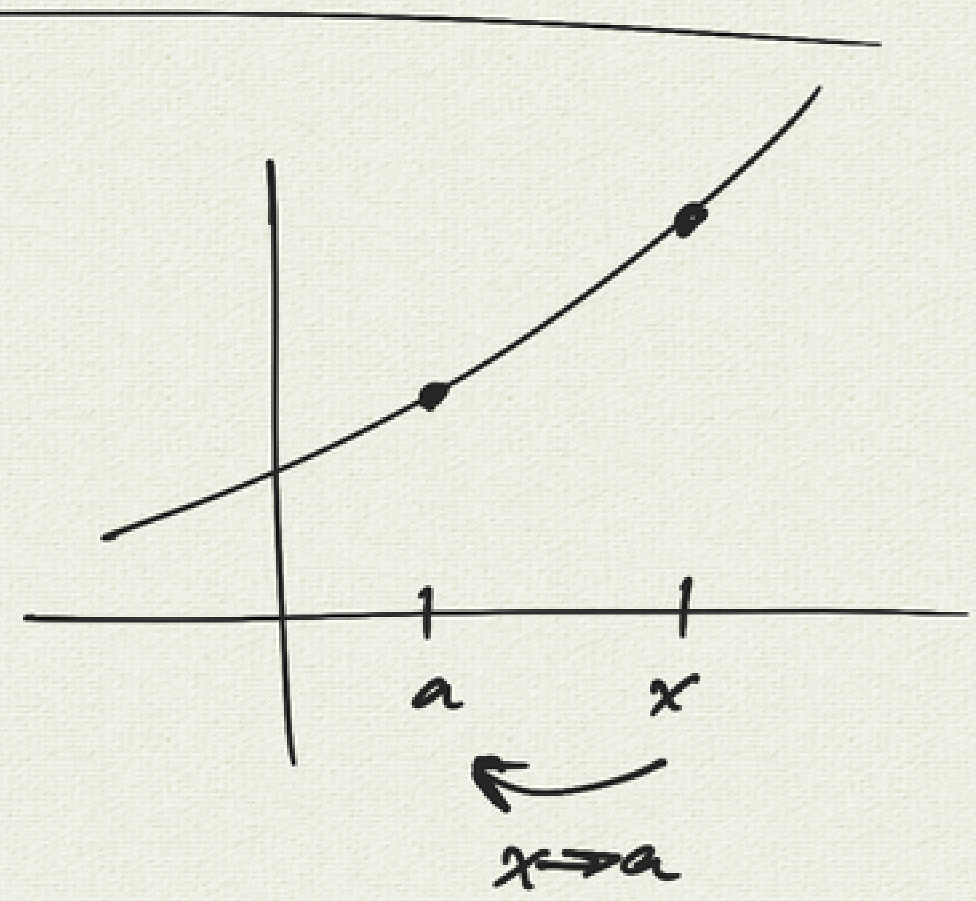
$$=\lim_{x\to a}\frac{-1}{ax}$$

a=3

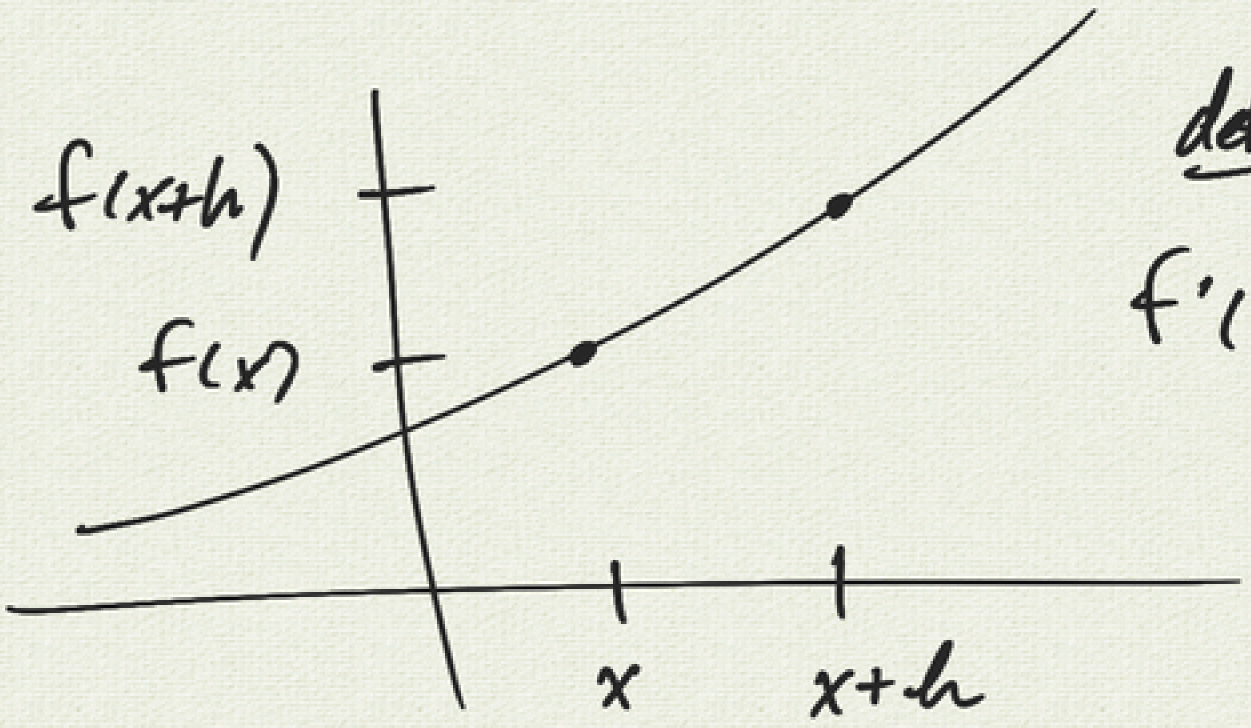
$$=\lim_{x\to 3}-\frac{1}{3x}$$

$$=-\frac{1}{9}$$





another view:



define:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

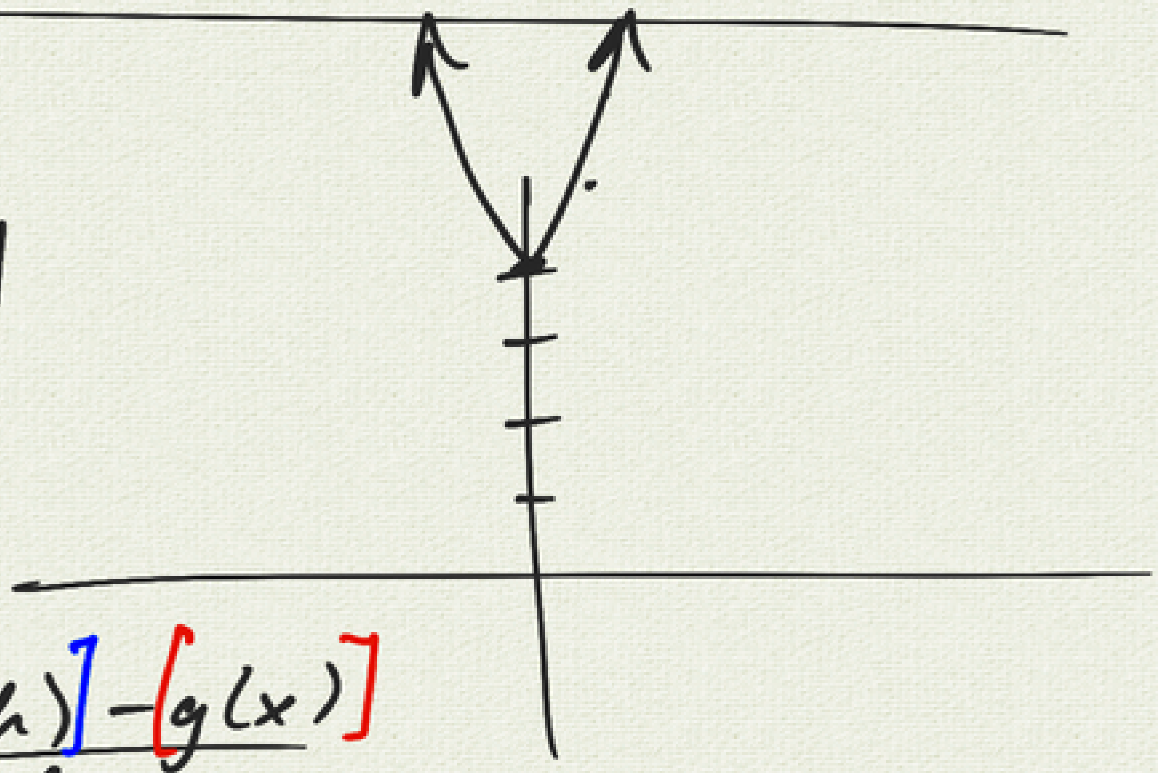
slope of tangent line at x

"the derivative of f at x"

example:

$$g(x) = 2x^2 + 4$$

find $g'(x)$



$$g'(x) = \lim_{h \to 0} \frac{[g(x+h)] - [g(x)]}{h}$$

$$= \lim_{h \to 0} \frac{[2(x+h)^2 + 4] - [(2x^2 + 4)]}{h}$$

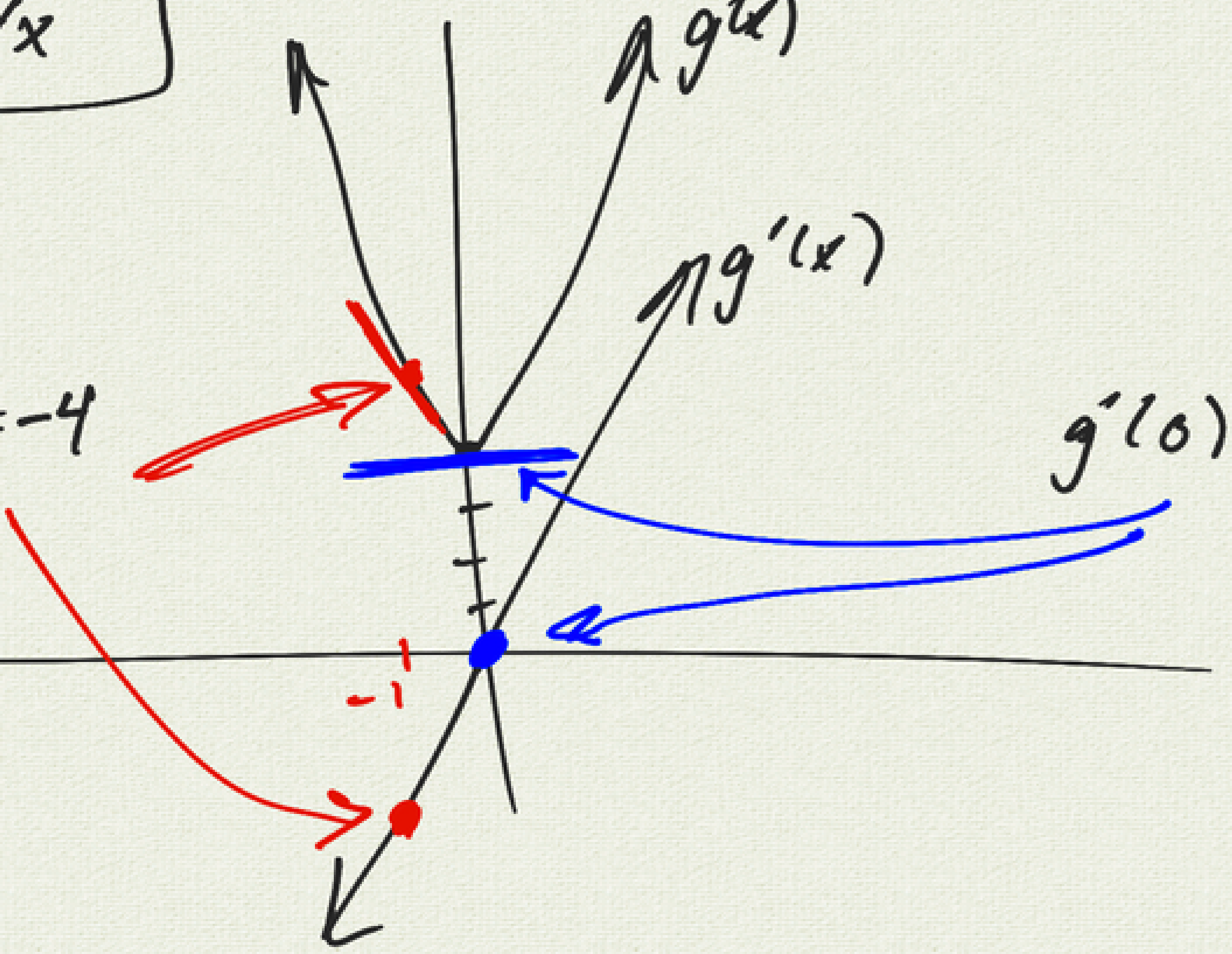
$$= \lim_{h \to 0} \frac{2(x^2 + 2xh + h^2) + 4 - (2x^2 + 4)}{h}$$

$$= \lim_{h \to 0} \frac{4xh + 2h^2}{h}$$

$$= \lim_{h \to 0} 4x + 2h$$

$$\boxed{g'(x) = 4x}$$

$g'(-1) = -4$



$g'(0) = 0$