(113) $f(x) = x^2\left(\frac{2}{x^2} + \frac{5}{x^3}\right)$

(1) multiply first

$$\boxed{f(x) = 2 + \frac{5}{x}}$$

$$= 2 + 5x^{-1}$$

$$\Rightarrow f'(x) = 5(-1 \cdot x^{-2}) \qquad \text{(power rule)}$$

$$= \frac{-5}{x^2}$$

---

(2) product rule

$$f(x) = \underbrace{x^2}_{g}\underbrace{\left(\frac{2}{x^2} + \frac{5}{x^3}\right)}_{h}$$

$(g \cdot h)' = g'h + gh'$

$x^n \qquad nx^{n-1}$

$h(x) = 2\boxed{x^{-2}} + 5\boxed{x^{-3}}$

$h'(x) = 2\boxed{-2x^{-3}} + 5\boxed{-3x^{-4}}$

$= \frac{-4}{x^3} - 15/x^4$

$$= (2x)\left(\frac{2}{x^2} + \frac{5}{x^3}\right) + x^2\left(\frac{-4}{x^3} - \frac{15}{x^4}\right)$$

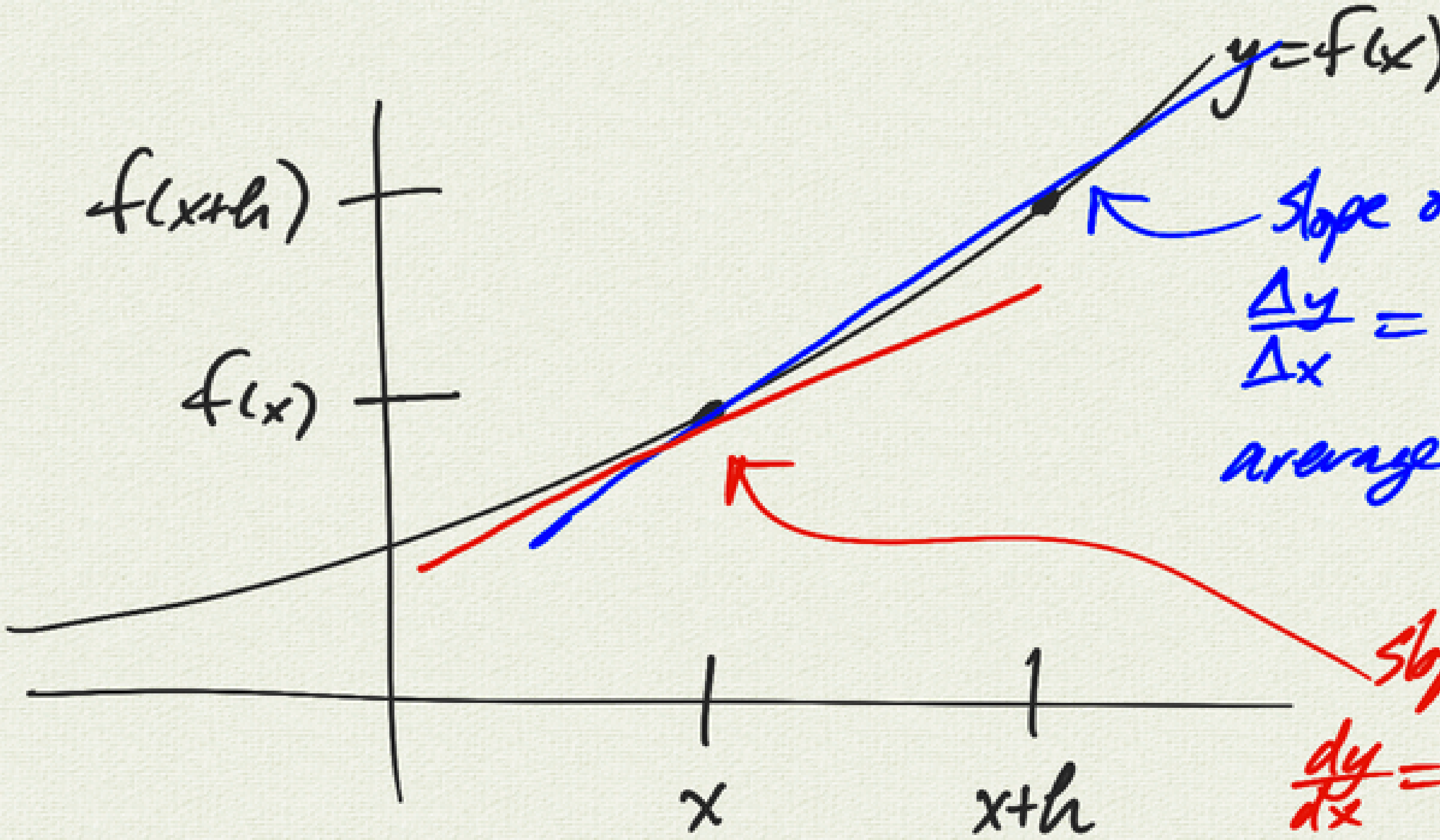
$$= \underline{\underline{\frac{4}{x}}} + \frac{10}{x^2} - \underline{\underline{\frac{4}{x}}} - \frac{15}{x^2}$$

$$= -\frac{5}{x^2} = -5x^{-2}$$

power rule

$$\boxed{\frac{d}{dx}(x^n) = nx^{n-1} \quad n \in \mathbb{Z}}$$

8.5 Rates of Change



slope of secant

$$\frac{\Delta y}{\Delta x} = \frac{f(x+h)-f(x)}{h}$$

average rate of change

slope of tangent

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

instantaneous rate of change

derivative

<u>notation:</u>

$f'(x) \quad \frac{dy}{dx} \quad \frac{df}{dx}$

function

$\Rightarrow$ 2nd derivative $f''(x) = \left(f'(x)\right)'$

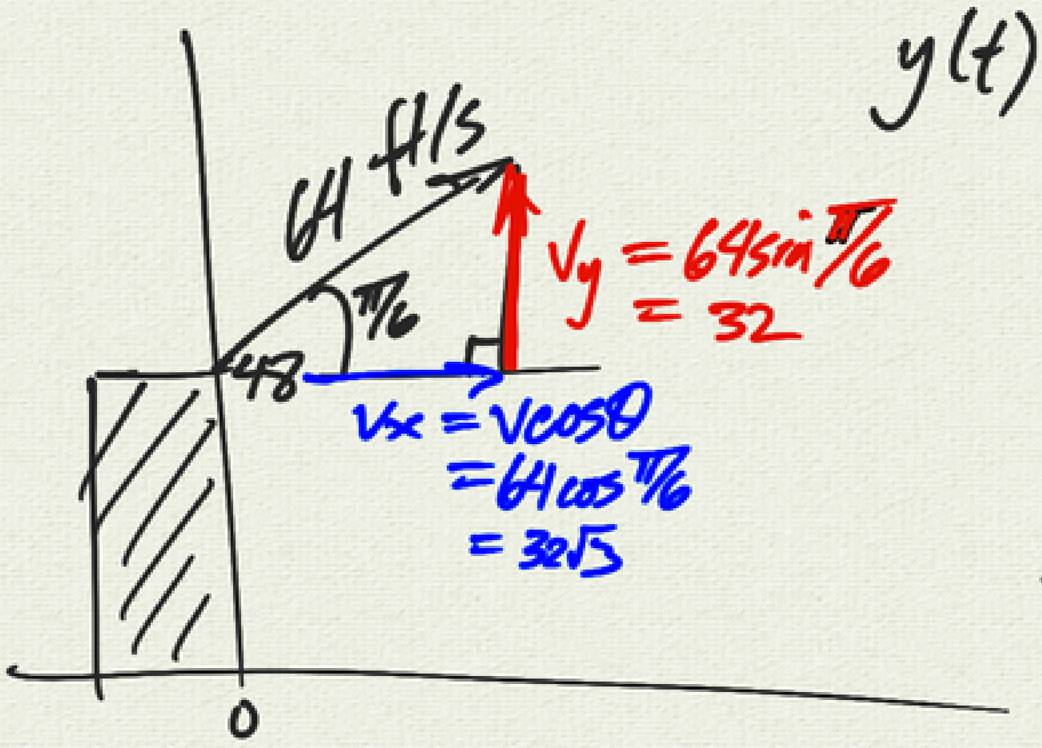
notation: $\frac{d^2 f}{dx^2} \quad \frac{d^2 y}{dx^2} \quad f''(x)$

derivative = rate of change

function f	f'	f''	f'''
position	velocity	acceleration	jerk

example: projectile motion

$$x(t) = x_0 + v_x t$$
$$y(t) = y_0 + v_y t - 16t^2$$



$$x(t) = 32\sqrt{3}\, t$$
$$y(t) = -16t^2 + 32t + 48$$

$$\frac{d}{dt}(t^n) = nt^{n-1}$$

$$x(t) = 32\sqrt{3}\, t$$
$$\Rightarrow \quad x'(t) = \frac{dx}{dt} = 32\sqrt{3}$$ x velocity is constant

$$x''(t) = \frac{d^2x}{dt^2} = 0$$ x acceleration is 0 (no force)

$$y(t) = -16t^2 + 32t + 48$$

$$\frac{d}{dt}(t^2) = 2t$$

$$\Rightarrow \quad y'(t) = \frac{dy}{dt} = \underbrace{-32t + 32}_{\text{y velocity}}$$

starts at 32 ft/s

-32 ft/s^2

$$y''(t) = \frac{d^2y}{dt^2} = -32$$ y acceleration is constant -32 ft/s^2

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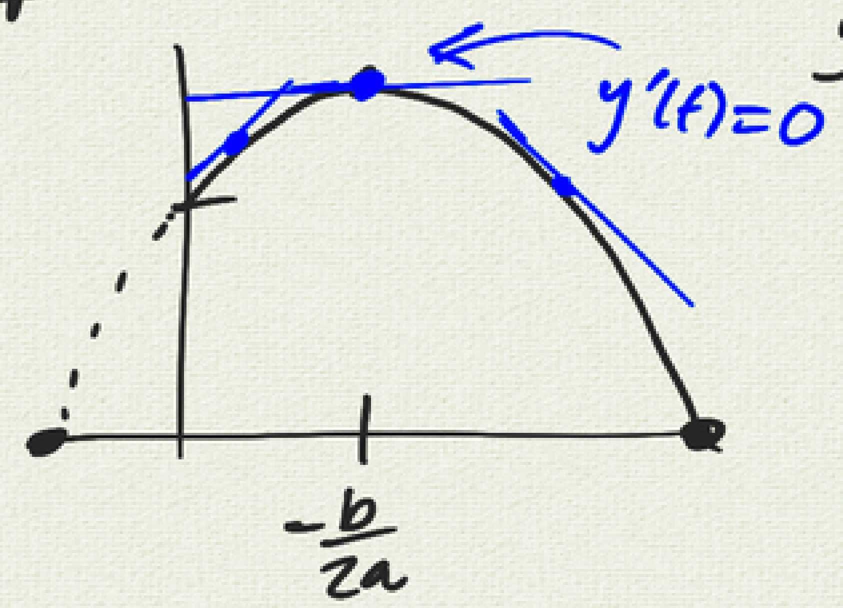
question: what is max height?

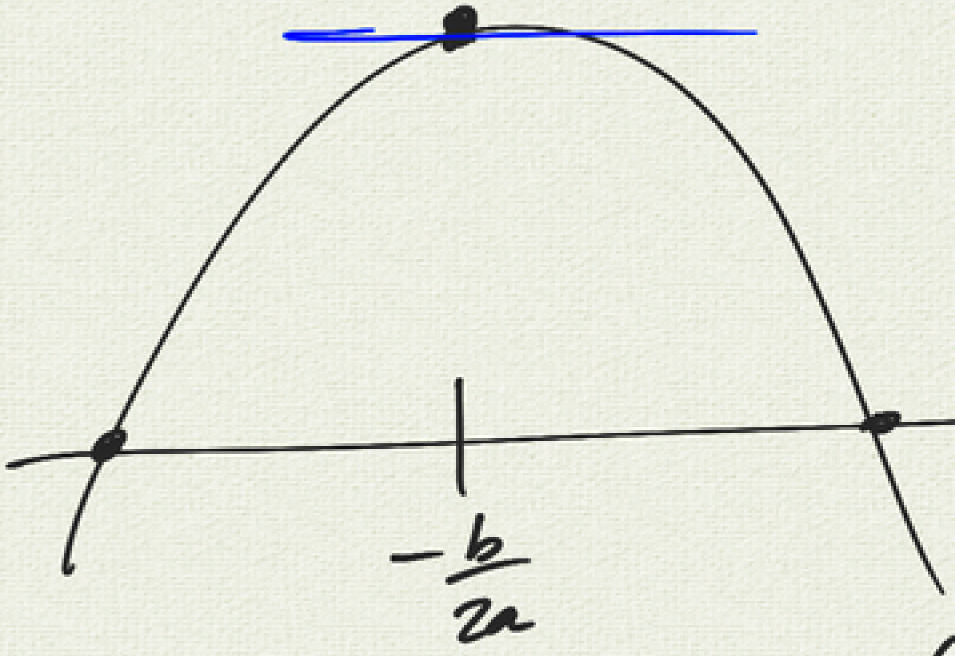
$$y(t) = -16t^2 + 32t + 48$$



$$y'(t) = -32t + 32$$

$$y'(t) = 0 \Rightarrow -32t + 32 = 0$$
$$t = 1$$

max height
$$y(1) = -16 + 32 + 48$$
$$= 64$$



quadratic

$$f(x) = ax^2 + bx + c$$

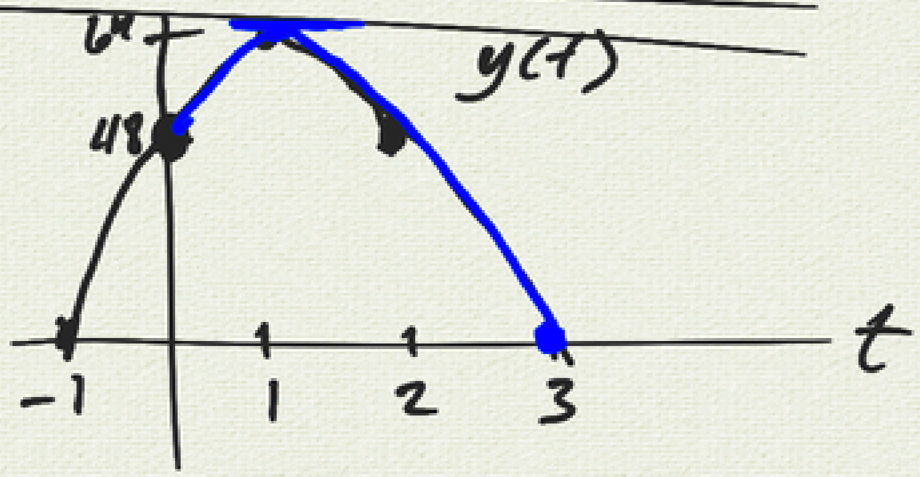
vertex $\Leftarrow$ where $f'(x) = 0$

$$f'(x) = 2ax + b$$

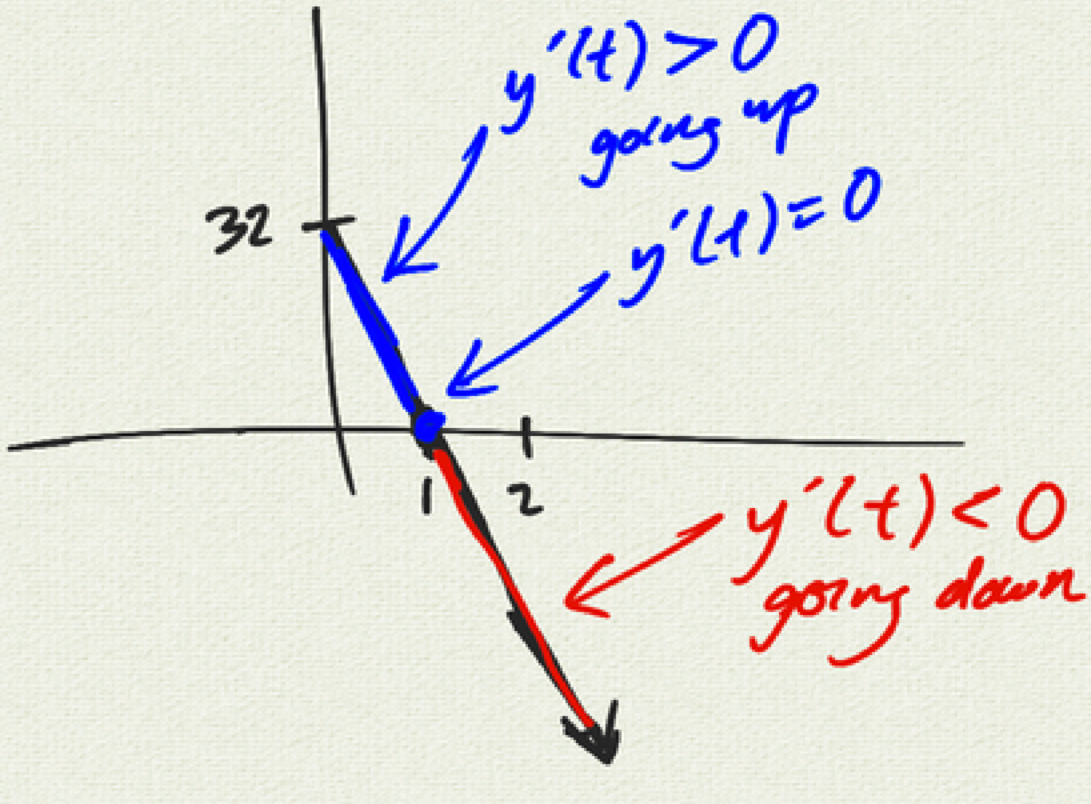
$$f'(x) = 0 \Rightarrow 2ax + b = 0$$

$$x = -\frac{b}{2a}$$

$$y(t) = -16t^2 + 32t + 48$$
$$= -16(t^2 - 2t - 3)$$
$$= -16(t-3)(t+1)$$



$$y'(t) = -32t + 32$$



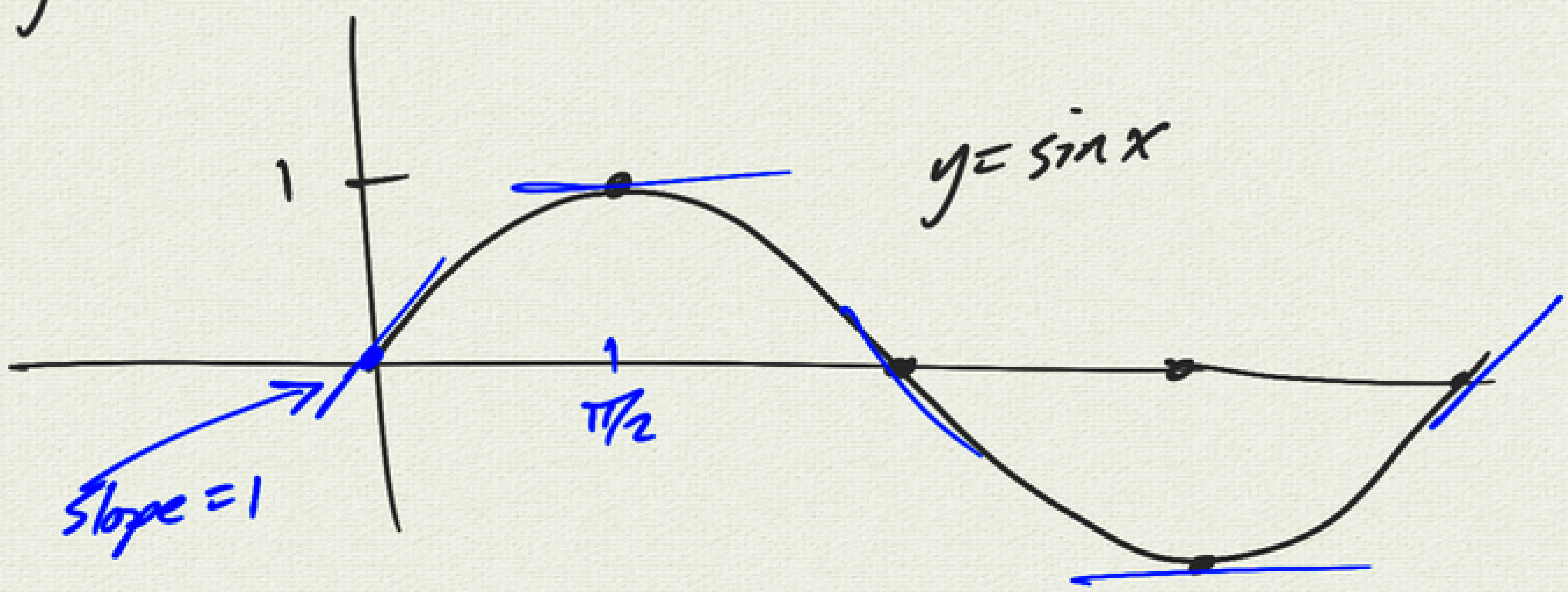
<u>recap:</u> $f(t)$ position

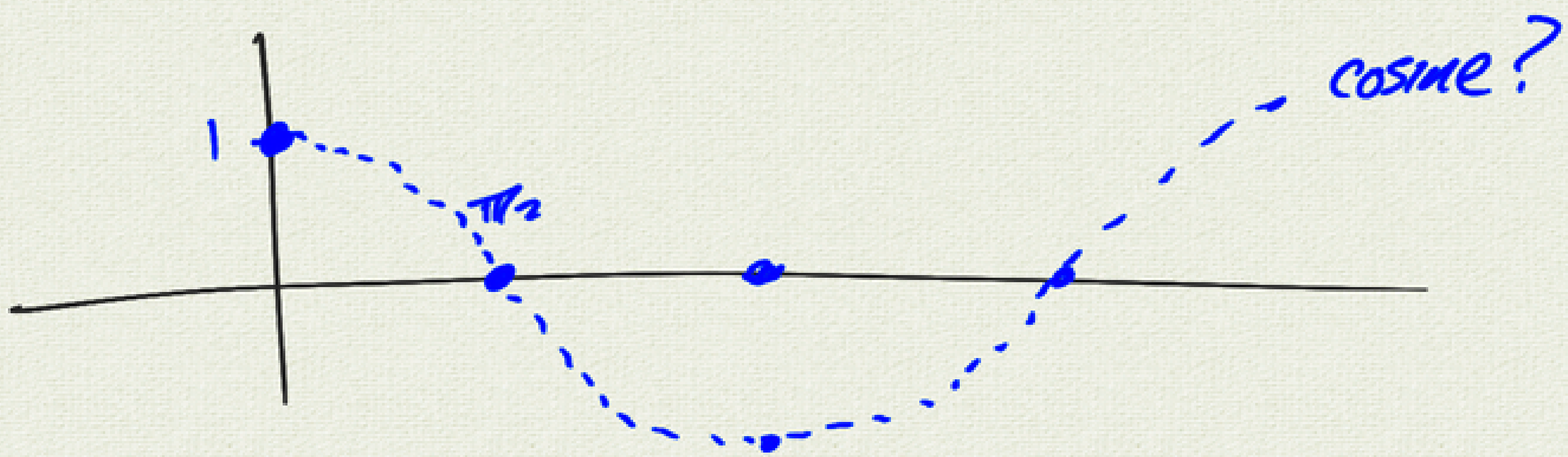
$f'(t) = \frac{df}{dt}$ velocity / rate of change

slope of tangent line

$f''(t) = \frac{d^2f}{dt^2}$ acceleration

$y = \sin x$





rules summary

$f(x) = \text{const} \Rightarrow f'(x) = 0$ $\quad \frac{d(\text{const})}{dx} = 0$

$(f+g)'(x) = f'(x) + g'(x)$ $\quad \frac{d(f+g)}{dx} = \frac{df}{dx} + \frac{dg}{dx}$

$(kf)'(x) = kf'(x)$ $\quad \frac{d}{dx}(kf) = k\frac{df}{dx}$

$f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$ (power rule $n \in \mathbb{Z}$) $\quad \frac{d(x^n)}{dx} = nx^{n-1}$

$(fg)'(x) = f'g + fg'$ (product rule) $\quad \frac{d(fg)}{dx} = \frac{df}{dx}g + f\frac{dg}{dx}$

Special limits:

$$\lim_{x \to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$$

$\frac{df}{dx} =$ derivative of f $= f'(x)$

$\frac{d(f)}{dx}$

$\frac{d(x^2)}{dx} = 2x$

$\frac{dy}{dx}$