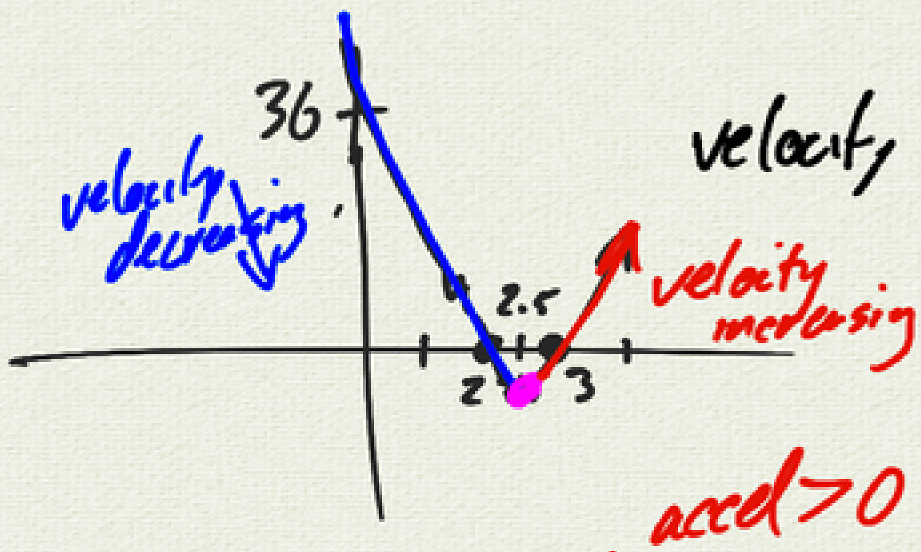
(151) $s(t) = 2t^3 - 15t^2 + 36t - 10$

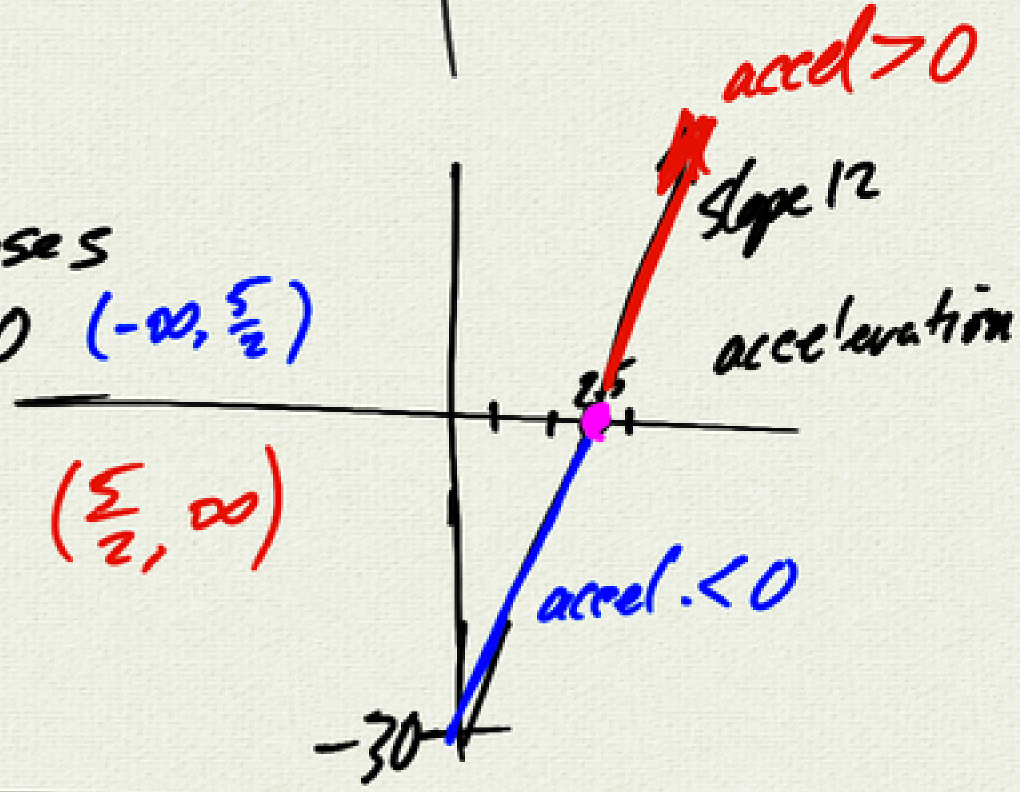
$s'(t) = 6t^2 - 30t + 36$ velocity

$s''(t) = 12t - 30$ acceleration

$s'(t) = 6(t^2 - 5t + 6)$
$= 6(t-2)(t-3)$

$s''(t) = 6(2t-5)$





---

slow down = velocity decreases
acceleration < 0 $(-\infty, \frac{5}{2})$

---

speed up = velocity increases
acceleration > 0 $(\frac{5}{2}, \infty)$

---

acceleration > 0 $\Rightarrow$ rate of change of velocity > 0 $\Rightarrow$ velocity is increasing

$\frac{d^2x}{dt^2} = \frac{d}{dt}\Big(\underbrace{\frac{dx}{dt}}_{\text{velocity}}\Big)$

slope > 0

velocity > 0 $\Rightarrow$ rate of change of position > 0 $\Rightarrow$ position increasing

$\frac{dx}{dt} > 0$

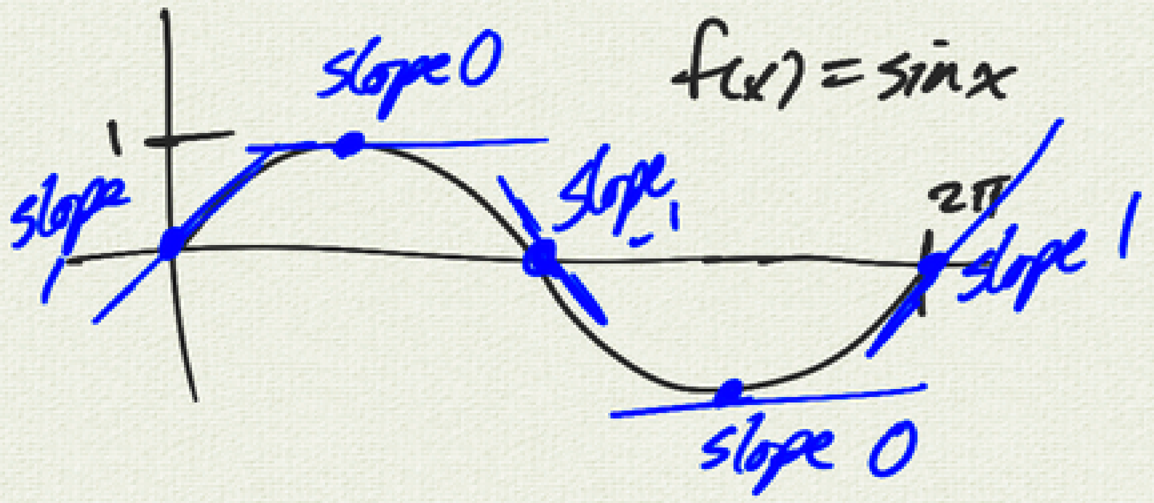
slope > 0

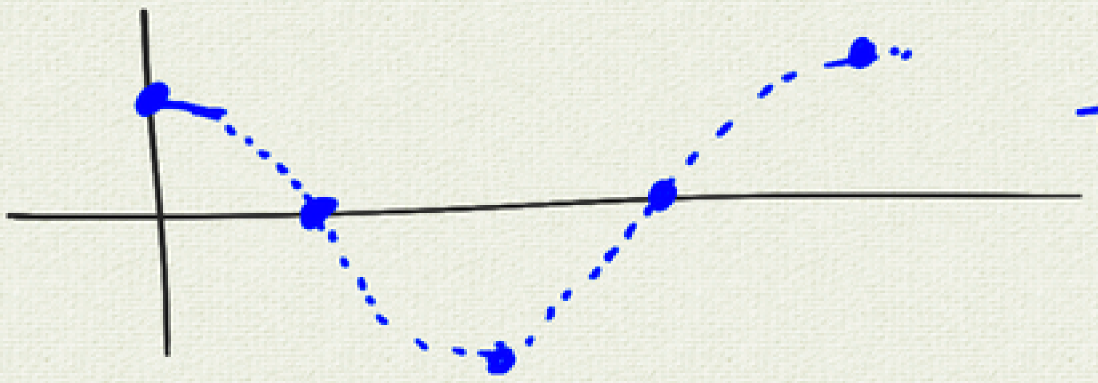
derivative > 0 $\Rightarrow$ rate of change of function > 0 $\Rightarrow$ function increasing

$f'(x) = \frac{df}{dx} > 0$

slope > 0

8.6 Trig Functions





guess:

$f'(x) = \cos x$

$\frac{d}{dx}(\sin x) = \cos x$

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \sin x \underbrace{\frac{(\cos h - 1)}{h}}_{\to 0} + \cos x \underbrace{\frac{\sin h}{h}}_{\to 1}$$

$$= \cos x$$

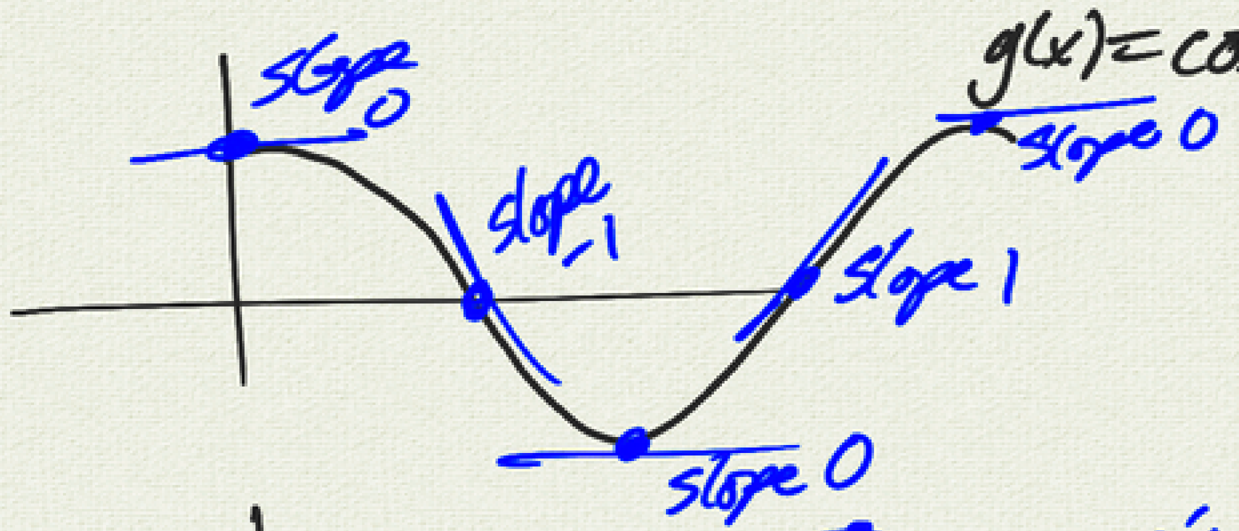
$\sin(u+v)$
$= \sin u \cos v + \cos u \sin v$

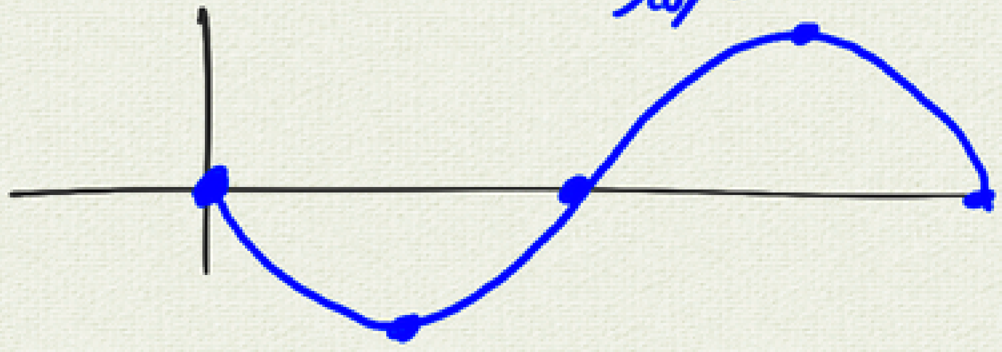
special limits:

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{\cos x - 1}{x} = 0$$

$$\boxed{\frac{d}{dx}(\sin x) = \cos x}$$





$g'(x) = -\sin x$

(exercise: do this with limit definition) example:

$$h(x) = 5\sin x + 3\cos x + x^5$$

$$\Rightarrow h'(x) = 5\cos x - 3\sin x + 5x^4$$

$$\tan x = \frac{\sin x}{\cos x}$$

quotient rule:

$$(f/g)' = \frac{f'g - fg'}{g^2}$$

$$\Rightarrow \frac{d}{dx}(\tan x) = \frac{(\cos x)\cdot\cos x - \sin x(-\sin x)}{\cos^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x}{\cos^2 x}$$

$$= \frac{1}{\cos^2 x}$$

$$\boxed{\frac{d}{dx}(\tan x) = \sec^2 x}$$

$$\frac{d}{dx}(\sec x) = \frac{d}{dx}\left(\frac{1}{\cos x}\right)$$

$$= \frac{0\cdot\cos x - 1\cdot(-\sin x)}{\cos^2 x}$$

$$= \frac{\sin x}{\cos^2 x}$$

$$\boxed{\frac{d}{dx}(\sec x) = \sec x \tan x}$$

$\boxed{\frac{d}{dx}(\quad)}$ = the derivative of

$\frac{df}{dx}$ = derivative of f

$$\cos^2 x = (\cos x)^2$$

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d(\cos x)}{dx} = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$

$\frac{d}{dx}(\csc x) = -\csc x \cot x$

---

example:

$$f(x) = 2\sin x \cos x$$

$$f'(x) = 2\left[(\cos x)(\cos x) + \sin(-\sin x)\right]$$

$$= 2\underbrace{(\cos^2 x - \sin^2 x)}_{\cos 2x}$$

another way: $f(x) = \sin 2x$

in the future

$\Rightarrow$?

$$f'(x) = 2\cos 2x$$

product rule:

$(fg)' = f'g + fg'$

$$\boxed{\cos^2 x + \sin^2 x = 1}$$

sum angle

$\sin(u+v) = \sin u \cos v + \cos u \sin v$

$\sin 2u = \sin(u+u)$

$= 2\sin u \cos u$

double angle