(1b)

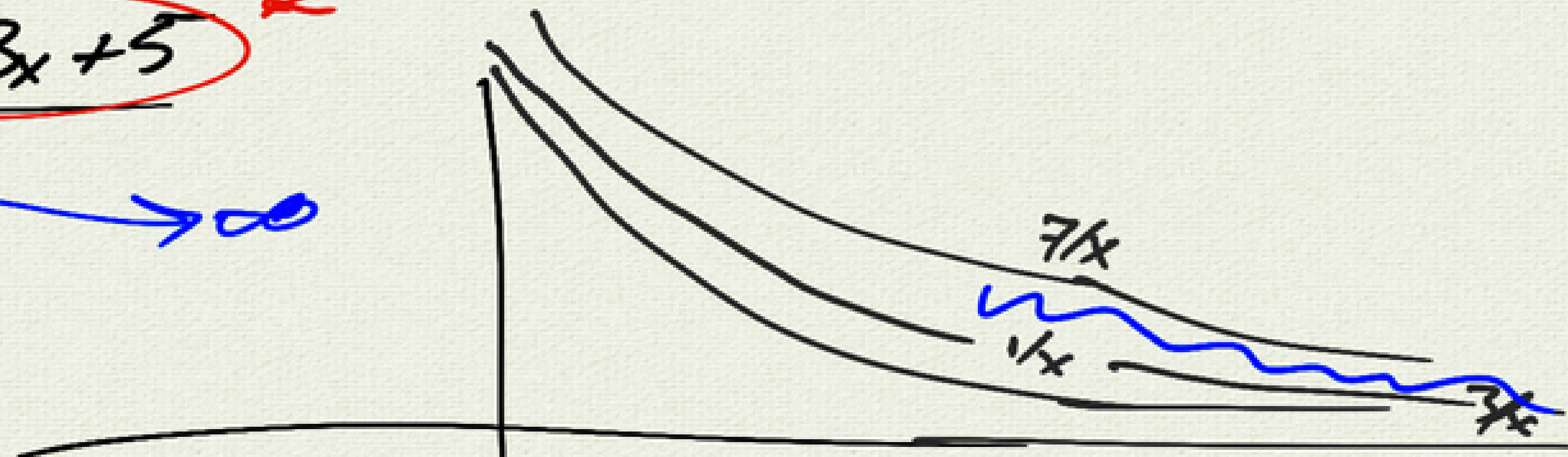
$[-1, +1]$ $[-1, +1]$

$3 \le \text{numerator} \le 5$

$$\lim_{x\to\infty} \frac{\sin 2x + \cos 3x + 5}{x}$$

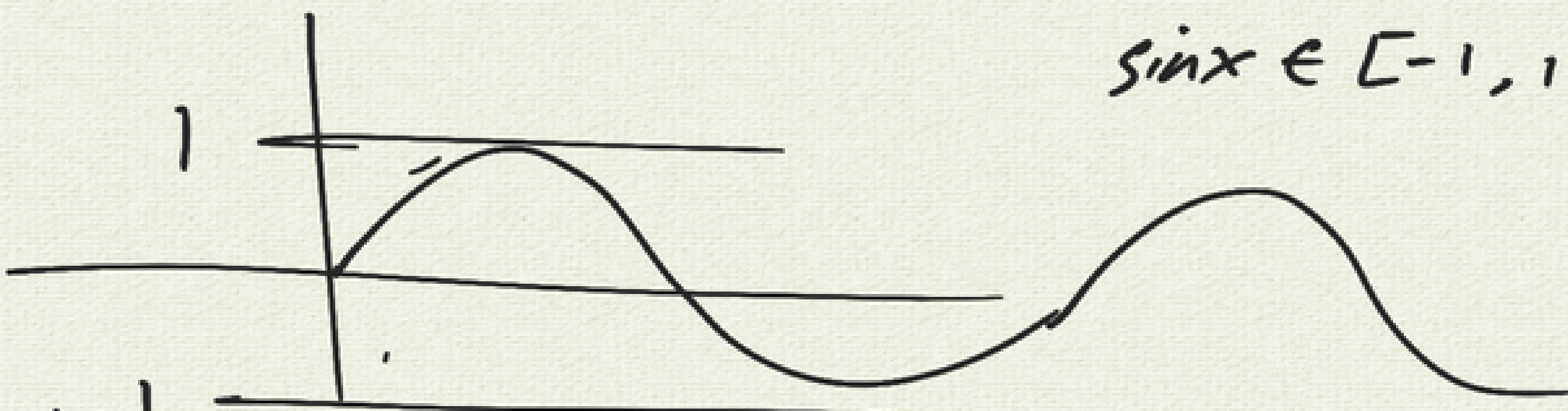
$x \to \infty$

$= 0$



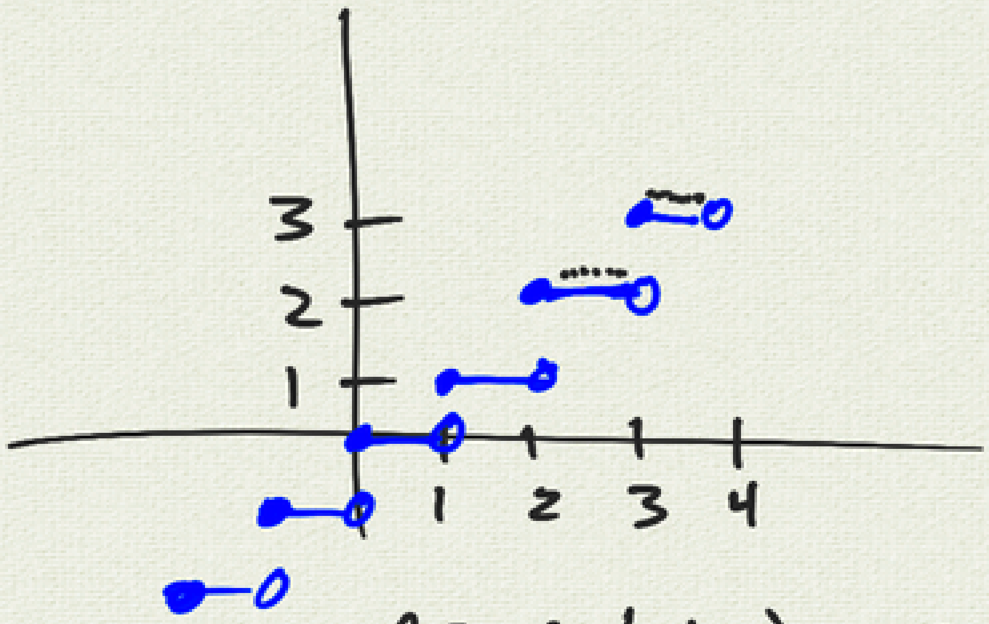
$$\frac{3}{x} \le \frac{\sin 2x + \cos 3x + 5}{x} \le \frac{7}{x}$$

$\sin x \in [-1, 1]$



(c) int(x)

$$\lim_{x\to 3} \text{int}(x)$$



int(3) = 3
int(3.1) = 3
int(3.01) = 3

---

int(2.9) = 2

$$\left.\begin{array}{l} \lim_{x\to 3^-} \text{int}(x) = 2 \\ \lim_{x\to 3^+} \text{int}(x) = 3 \end{array}\right\}$$ $\lim_{x\to 3} \text{int}(x)$ does not exist

---

(H) $$\lim_{x\to -5} \frac{(\cancel{x+5})(x-3)}{(\cancel{x+5})(x-7)}$$

$$= \lim_{x\to -5} \frac{x-3}{x-7} = \frac{-8}{-12}$$

$\frac{0}{0} \longrightarrow$? $\Longrightarrow$ cancel or special limit

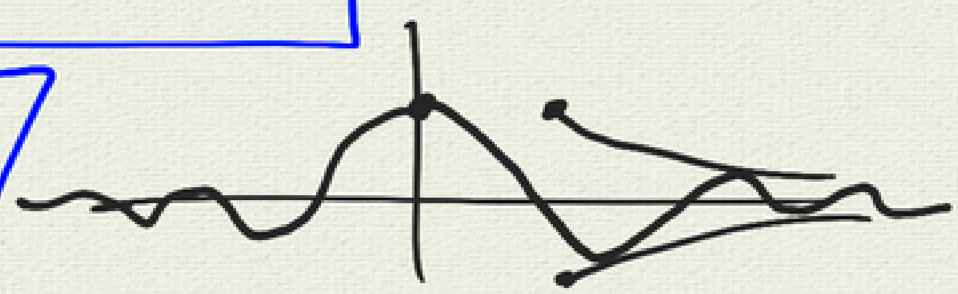
I don't know

<u>special limits:</u>

$$\boxed{\lim_{x\to 0} \frac{\sin x}{x} = 1}$$

$\sin x \approx x$ as $x \to 0$

$$\boxed{\lim_{x\to 0} \frac{\cos x - 1}{x} = 0}$$



(1d) $\lim\limits_{x\to 0} \dfrac{\sin 5x}{5x} = 1$

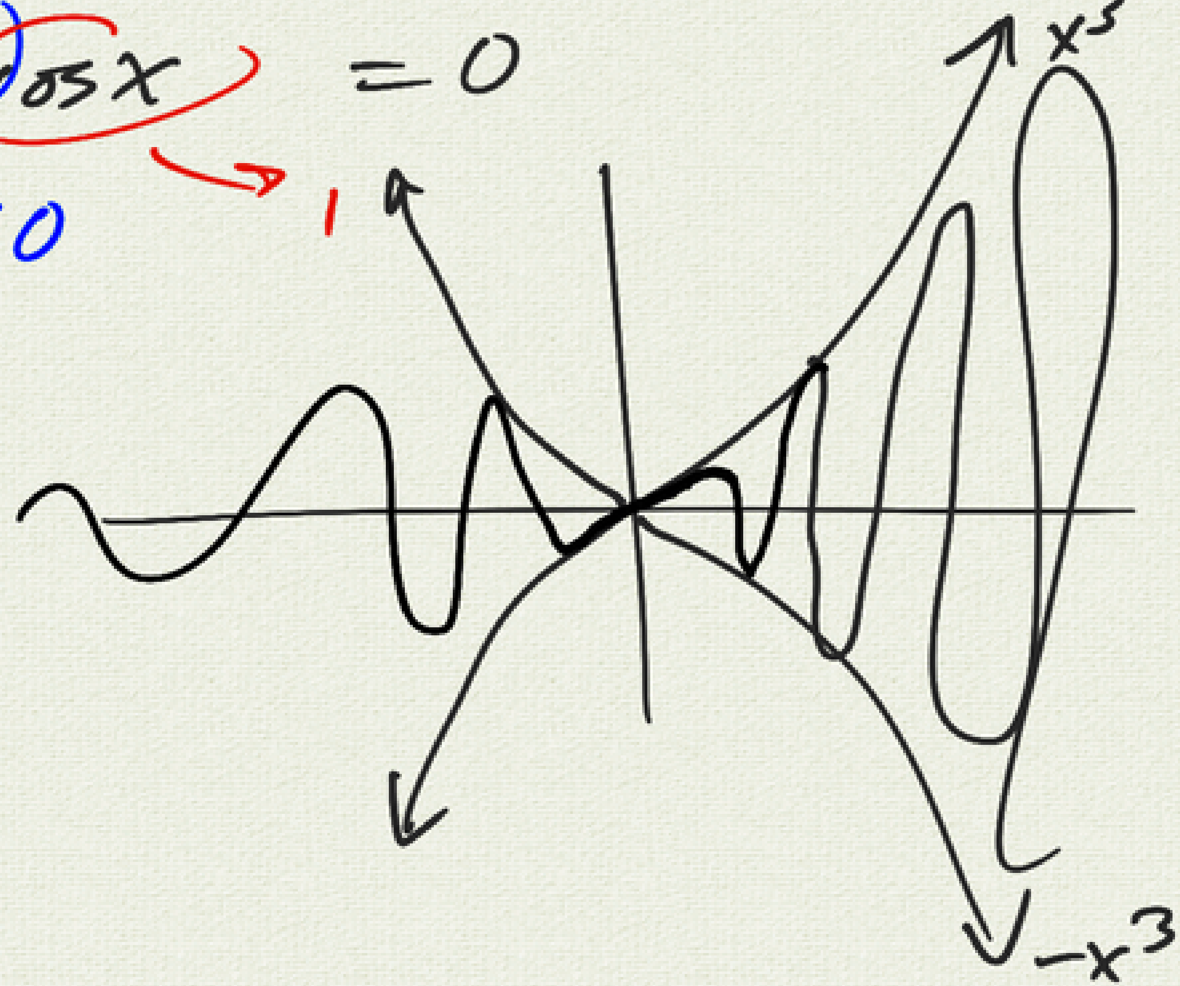
$5x \to 0$

$\sin 5x \approx 5x$ as $x \to 0$

$\sin x \approx x$

(1e) $\lim\limits_{x\to 0} \dfrac{\sin 5x}{x} \cdot \dfrac{5}{5} = 5$

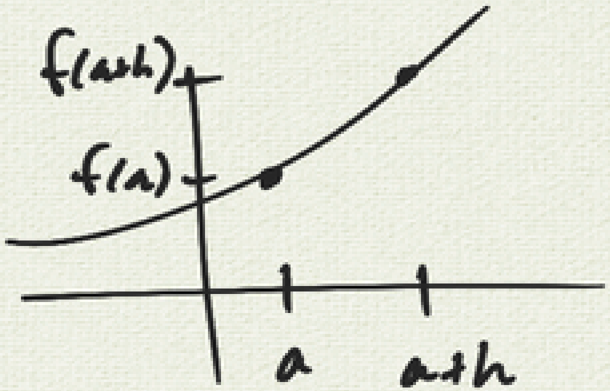
(1a) $\lim\limits_{x\to 0} \underbrace{x^3}_{\to 0}\,\underbrace{\cos x}_{\to 1} = 0$



$\boxed{x^3}\cos x$

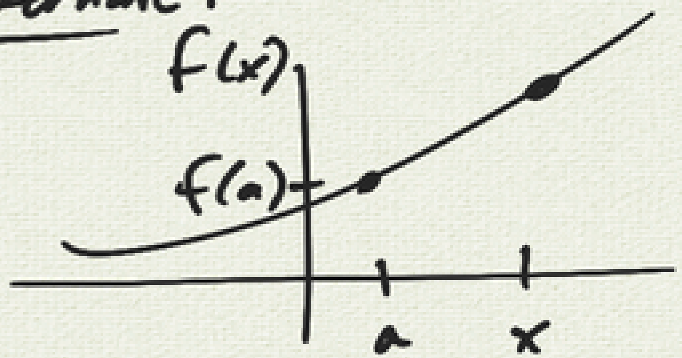
$\uparrow$ amplitude

3b



$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

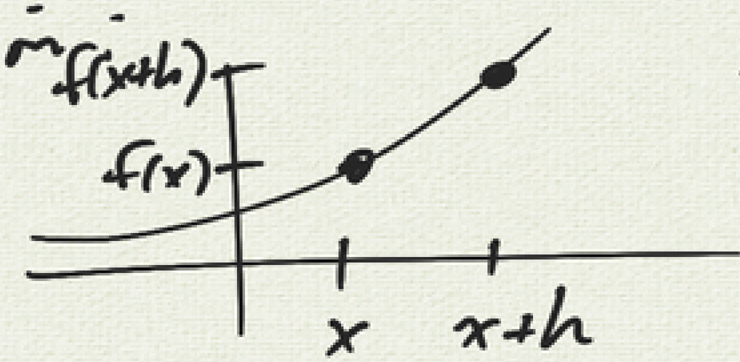
alternate:



$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

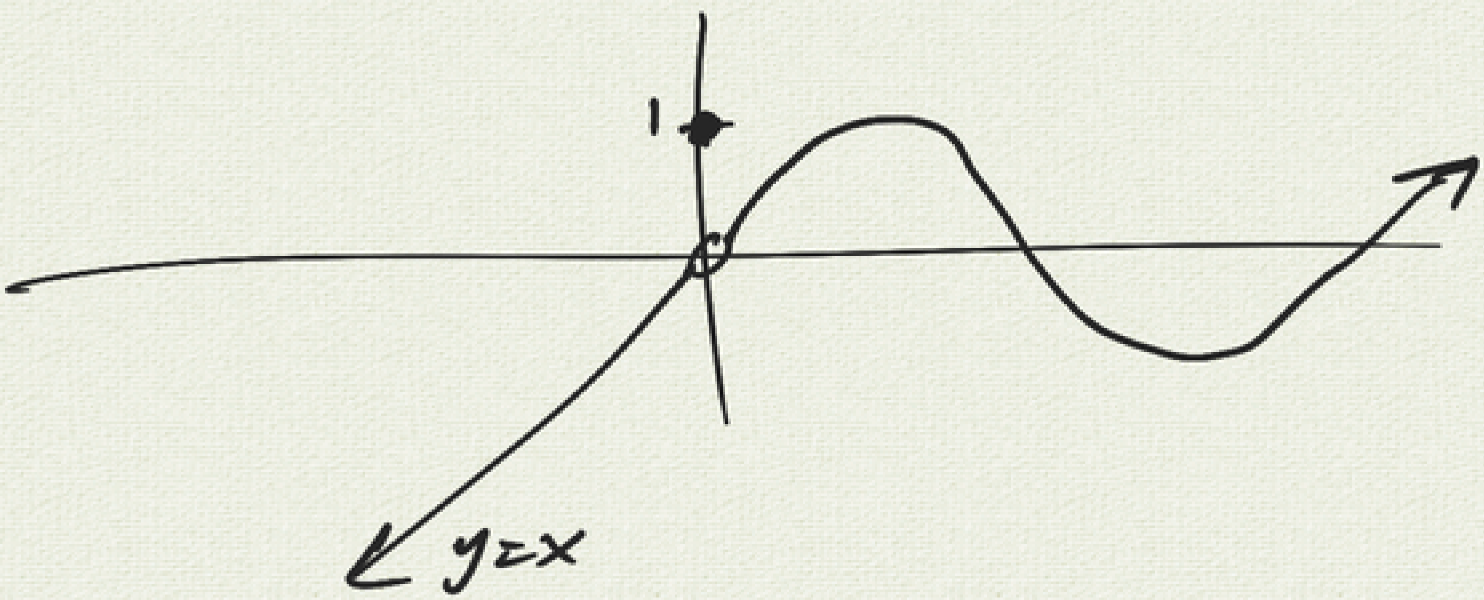
---

function

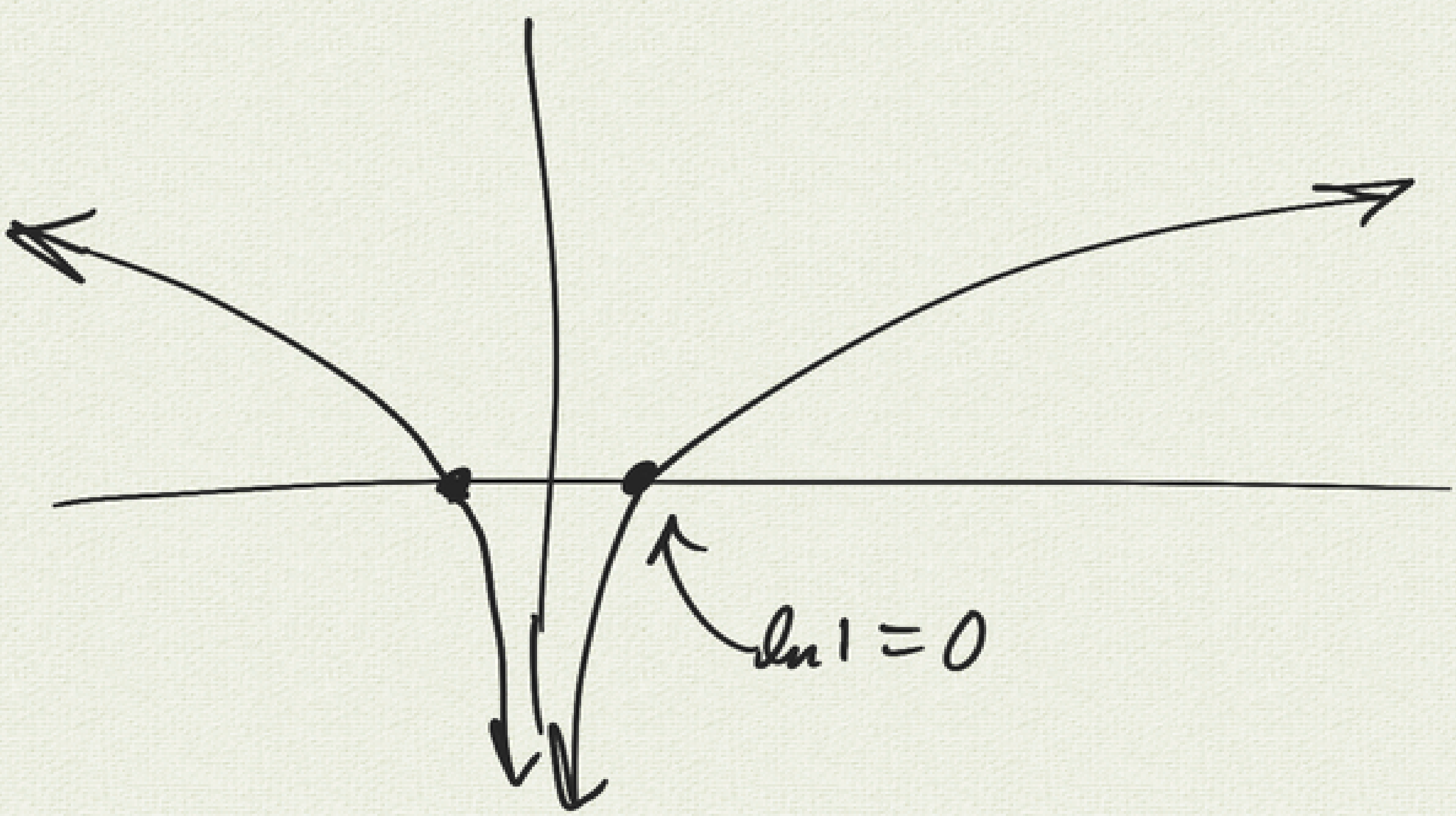


$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

② $g(x) = \begin{cases} x & \text{if } x<0 \\ 1 & \text{if } x=0 \\ \sin x & \text{if } x>0 \end{cases}$



$h(x) = \ln(|x|) \Leftarrow$ even $h(-x) = h(x)$



(5) $f(t) = t^3 - 4t$

$= t(t^2-4)$

$= t(t+2)(t-2)$

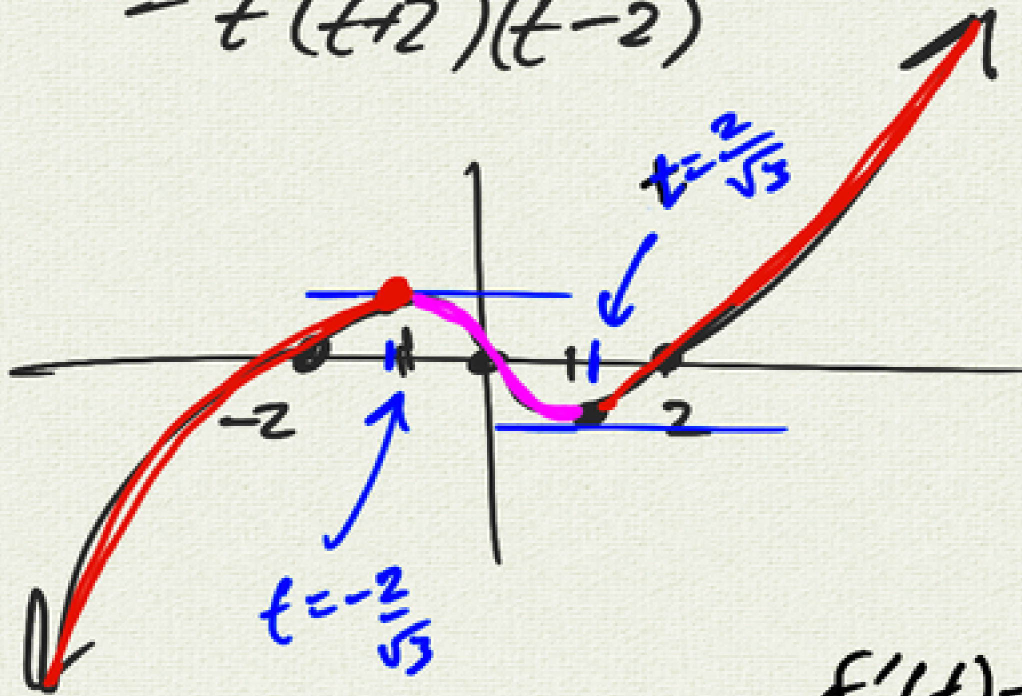
$f'(t) = 3t^2 - 4$

$f'(t) = 0$

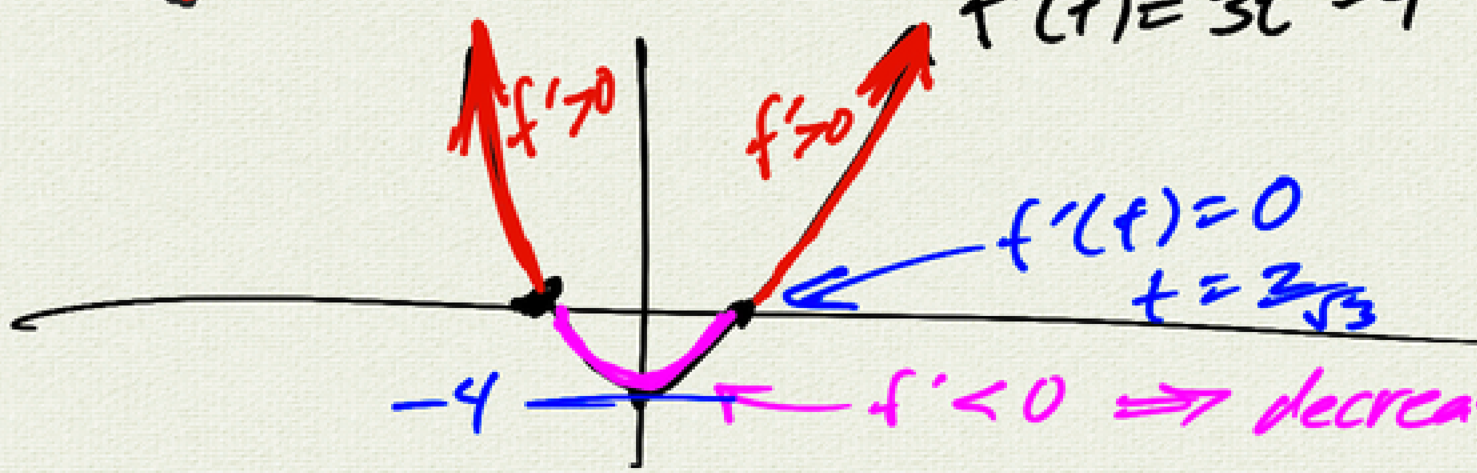
$\Rightarrow 3t^2 - 4 = 0$

$t^2 = \frac{4}{3}$

$t = \pm \frac{2}{\sqrt{3}}$

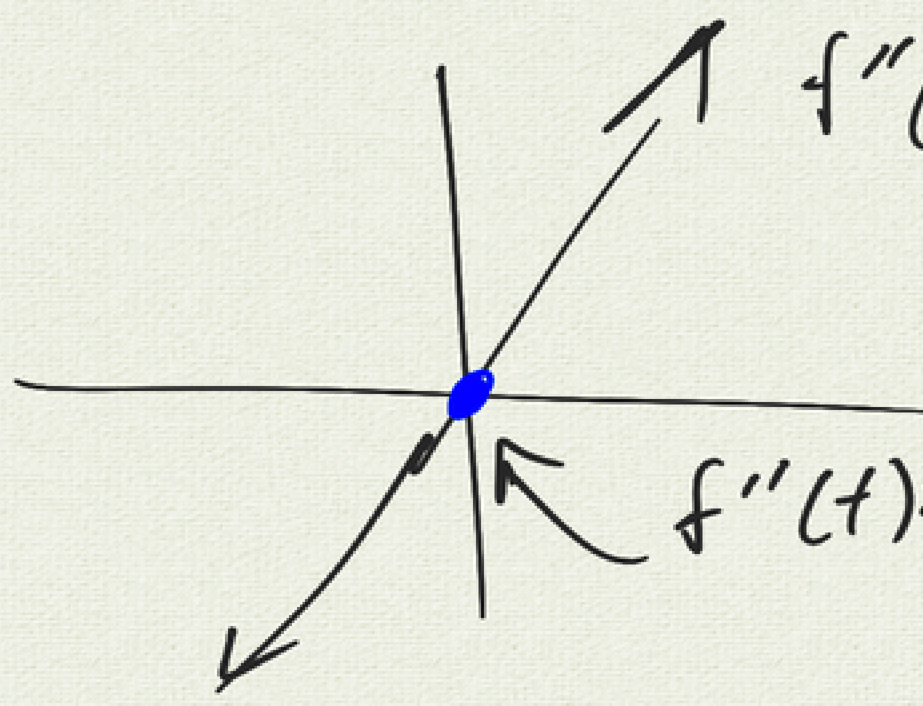


$f'(t) = 3t^2 - 4$



$f' < 0 \Rightarrow$ decreasing

$f''(t) = 6t$



$f''(t) = 0 \iff$ local min of $f'(t)$

9.1 Chain rule

$fg \Rightarrow$ product rule

$f/g \Rightarrow$ quotient rule

$f \circ g \Rightarrow$?

composition

f' = instantaneous rate of change

= slope

$$\#\text{hours} \xrightarrow{g} \#\text{coffees} \xrightarrow{f} \#\text{math problems}$$

$g' = 2$ — 2 coffees/hour

$f' = 10$ — 10 math problems/coffee

$$\Rightarrow \frac{\#\text{math problems}}{\text{hour}} = g' \cdot f'$$

rates multiply (chain rule)

Chain rule: $\boxed{(f \circ g)'(x) = f'(g(x)) \cdot g'(x)}$

example:

$$h(x) = \sin(x^2)$$

$$h'(x) = f'(g(x))\, g'(x)$$

$$= \cos(x^2) \cdot 2x$$

$$h = (f \circ g)(x)$$

$$= f(g(x))$$

$\Longrightarrow$ $f(x) = \sin x$ $\quad f'(x) = \cos x$

$g(x) = x^2$ $\quad g'(x) = 2x$

$$h(x) = \sin(x^2)$$

$$h'(x) = \cos(x^2) \cdot (2x)$$

$\frac{d}{dx}(\sin x) = \cos x$ $\quad \frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$ $\quad \frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$ $\quad \frac{d}{dx}(\csc x) = -\csc x \cot x$

$f(x) = (x^2+1)^2$ $\quad f(x) = (x)^2 \quad g(x) = x^2+1$

$\Rightarrow f'(x) = 2(x^2+1)\cdot(2x)$

derivative of inside

$g(x) = \cos(x^3+3x^2+2x+1)$

$g'(x) = -\sin(x^3+3x^2+2x+1)\cdot(3x^2+6x+2)$

$h(x) = \tan(5x^2+3x+2)$ $\quad \frac{d}{dx}(\tan x) = \sec^2 x$

$h'(x) = \sec^2(5x^2+3x+2)\cdot(10x+3)$

$$k(x) = \sin^4(x^5+2) = [\sin(x^5+2)]^4$$

$$k'(x) = 4[\underbrace{\sin(x^5+2)}]^3 \cdot \cos(\underbrace{x^5+2}) \cdot 5x^4$$

$$\frac{d}{dx}(x^4) = 4x^3$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$(f(gh))' = f'(gh) + f\,\underbrace{(gh)'}_{\text{product}}$$
