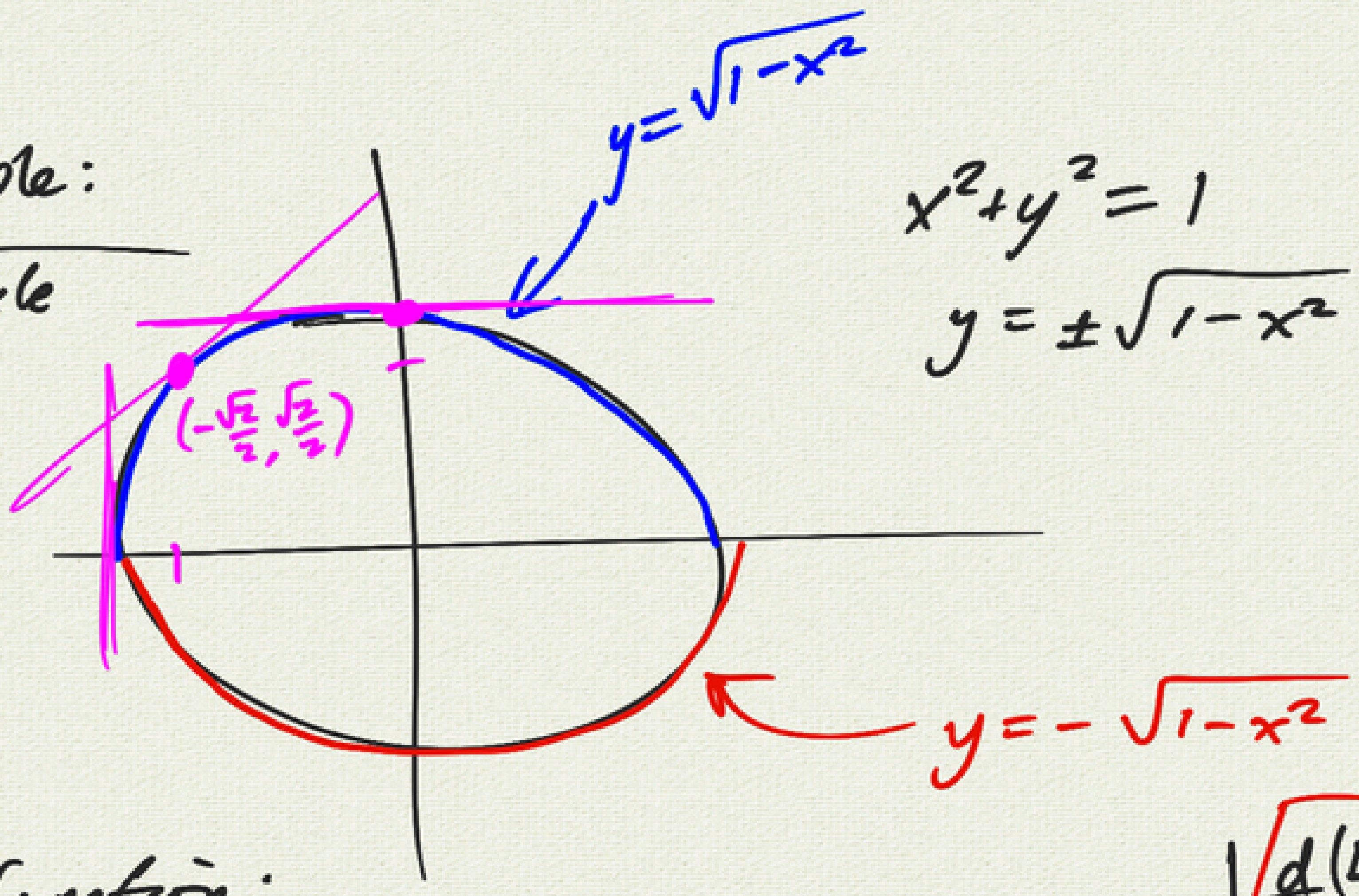
9.2 Implicit differentiation

power rule: $\frac{d}{dx}(x^n) = nx^{n-1}$ $\quad \frac{n \in \mathbb{Z}}{\text{integer}}$

example:
unit circle



$x^2+y^2=1$

$y=\pm\sqrt{1-x^2}$

$\frac{d(x^{\frac12})}{d} = \frac12 x^{-1/2}$

$$\boxed{\frac{d}{dx}(\square^{1/2}) = \frac12 \square^{-1/2} \cdot \square'}$$

$\frac{d}{dx}\left[(\sin x)^{1/2}\right] = \frac12 (\sin x)^{-1/2} \cdot \cos x$
chain rule

top function:

$f(x) = \sqrt{1-x^2} = (1-x^2)^{1/2}$

$\Rightarrow f'(x) = \frac12 (1-x^2)^{-1/2}(-2x)$

$= \frac{-x}{\sqrt{1-x^2}}$

check: $f'(0) = 0$ ✓

$f'(-1) = \text{undef}$

$f'(-\frac{\sqrt2}{2}) = \frac{+\sqrt2/2}{\sqrt{1-(\frac{\sqrt2}{2})^2}} = 1$ ✓

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another view:

implicit differentiation

$$\boxed{x^2+y^2=1}$$

take derivative

$$\boxed{2x + \left(2y\frac{dy}{dx}\right) = 0}$$

$\frac{dx}{dx}$ chain rule

$\frac{d}{dx}(\square^2) = 2\square \cdot \square'$

$\frac{d}{dx}(y^2) = 2y \cdot \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{-2x}{2y}$

$$\boxed{\frac{dy}{dx} = -\frac{x}{y}}$$

aside:
suppose $y = \sin x$ $\rightarrow \frac{dy}{dx} = \cos x$

what is derivative of $y^{1/2}$?

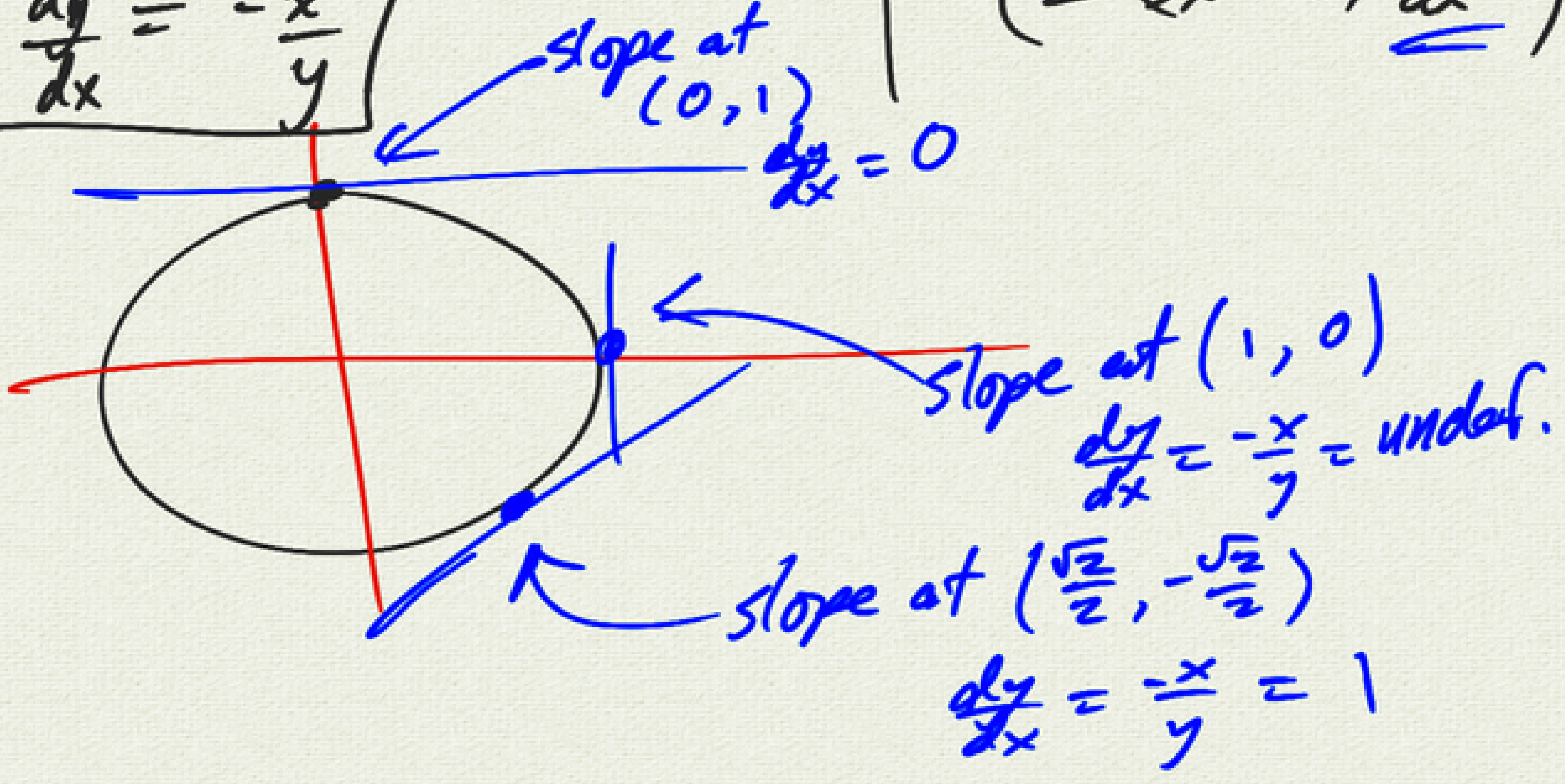
$$\boxed{\frac{d}{dx}(y^{1/2}) = \frac12 y^{-1/2} \cdot \frac{dy}{dx}}$$ different way of writing

$= \frac12 (\sin x)^{-1/2} \cdot \cos x$

$y = x^3+2x$ $\qquad x = x$

$\frac{dy}{dx} = 3x^2+2$ $\qquad \frac{dx}{dx} = 1$

$\left(= (3x^2+2)\frac{dx}{dx}\right)$



$y = f(x)$ explicit definition for y

$F(x,y) = 0$ implicit definition

$x^2+y^2-1=0$

example:

$$x^3 + 2x + y^3 + y^2 = 0$$

$\Rightarrow$ find $\frac{dy}{dx}$

(1) $3x^2 + 2 + 3y^2 \cdot \frac{dy}{dx} + 2y\frac{dy}{dx} = 0$

$$(3y^2 + 2y)\frac{dy}{dx} = -3x^2 - 2$$

(2) $\frac{dy}{dx} = \frac{-3x^2 - 2}{3y^2 + 2y}$

(1) differentiate both sides (with chain rule)

(2) solve for dy/dx

$$\frac{dx}{dx} = 1$$

power rule for fractions

$y = \sqrt[n]{x} = x^{1/n}$

$y^n = x$

$n y^{n-1} \cdot \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{n} \frac{1}{y^{n-1}}$ $y = x^{1/n}$

$= \frac{1}{n} \frac{1}{(x^{1/n})^{(n-1)}}$

$= \frac{1}{n} \frac{1}{x^{(1-1/n)}}$

$$\boxed{\frac{d}{dx}\left(x^{1/n}\right) = \frac{1}{n} x^{(1/n - 1)}}$$

power rule works for $x^{1/n}$

power rule for integers

chain rule / implicit differentiation

$\frac{d}{dx}(x^n) = n x^{n-1}$ $\underline{n \in \mathbb{Z}}$

$(x^a)^b = x^{ab}$

exercise:

$y = x^{p/q}$