307

$(fg)' = f'g + fg'$
product rule

$$y\sin(xy) = y^2 + 2$$

($f = y$, $g = \sin(xy)$)

$$\frac{dy}{dx}\cdot\sin(xy) + y\cdot\frac{d}{dx}[\sin(xy)] = 2y\frac{dy}{dx}$$

(f', g, f, g')

$$\frac{d}{dx}[\sin(xy)] = \cos(xy)\cdot\left(1\cdot y + x\frac{dy}{dx}\right)$$

chain rule: $\frac{d}{dx}(xy)$

$$\frac{dy}{dx}\sin(xy) + y^2\cos(xy) + xy\cos(xy)\frac{dy}{dx} = 2y\frac{dy}{dx}$$

$$y\cos(xy)\left[y + x\frac{dy}{dx}\right]$$

$$\frac{dy}{dx}\left[\sin(xy) + xy\cos(xy) - 2y\right] = -y^2\cos(xy)$$

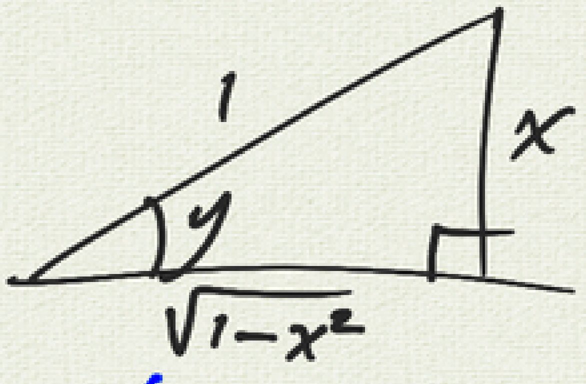
$$\frac{dy}{dx} = \frac{-y^2\cos(xy)}{[\sin(xy) + xy\cos(xy) - 2y]}$$

$$\frac{d}{dx}(x^n) = nx^{n-1}\left(\frac{dx}{dx}\right)$$

$$\frac{d}{dx}(y^n) = ny^{n-1}\frac{dy}{dx} \quad \Leftarrow \text{chain rule}$$

9.4 Inverse Trig

$y = \sin^{-1} x \Rightarrow \frac{dy}{dx} = ?$



$\Rightarrow \sin y = x$

$\cos y \frac{dy}{dx} = 1$

$\cos y = \sqrt{1-x^2}$

$\frac{dy}{dx} = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$

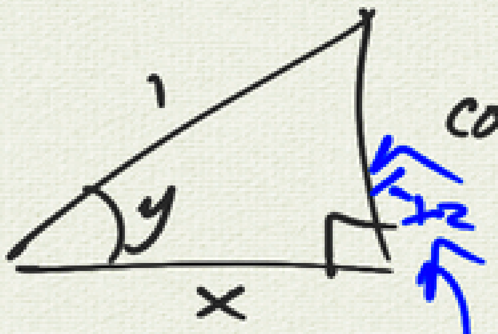
$$\boxed{\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}}$$

---

example: $f(x) = \sin^{-1}(x^3+3x)$

$f'(x) = \frac{1}{\sqrt{1-(x^3+3x)^2}} \cdot (3x^2+3)$

$y = \cos^{-1} x$



$\cos y = \frac{x}{1}$

$\Rightarrow \cos y = x$

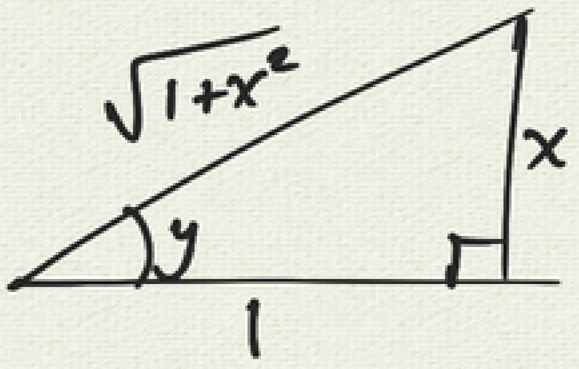
$-\sin y \frac{dy}{dx} = 1$

$\sin y$

$\frac{dy}{dx} = \frac{-1}{\sin y} = \frac{-1}{\sqrt{1-x^2}}$

$$\boxed{\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}}$$

$y = \tan^{-1} x$



$\Rightarrow \tan y = x$

$\sec^2 y \frac{dy}{dx} = 1$

$\frac{dy}{dx} = \frac{1}{\sec^2 y} = \cos^2 y$

$\tan y = \frac{x}{1}$

$\cos y = \frac{1}{\sqrt{1+x^2}}$

$$\boxed{\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}}$$

example:

$$g(x) = \tan^{-1}\left(e^{\sin x^2}\right)$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\Rightarrow g'(x) = \frac{1}{1+\left(e^{(\sin x^2)}\right)^2} \cdot e^{(\sin x^2)} (\cos x^2) \cdot 2x$$