

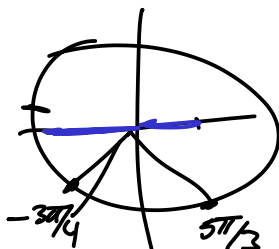
KEY

Unit 1 Test
PCHA 2020-21 / Dr. Kessner

No calculator, no notes – just your brain! Have fun!

1. Evaluate the following:

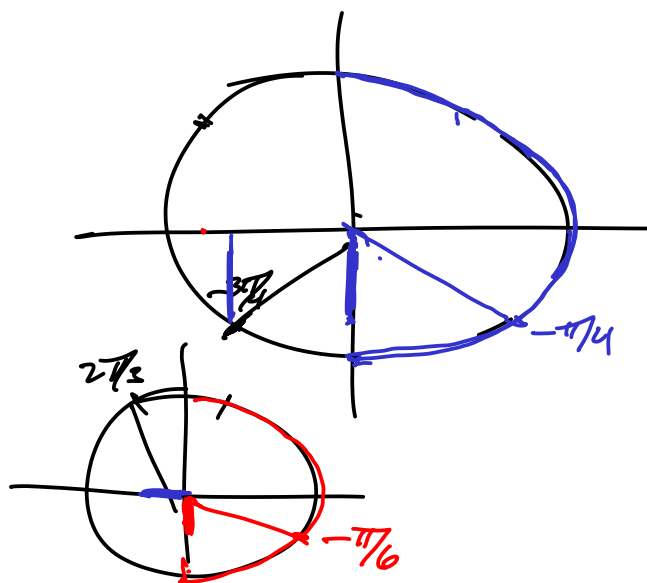
a) $\cos \frac{5\pi}{3} = \frac{1}{2}$



b) $\cot \left(-\frac{3\pi}{4}\right) = 1$

c) $\sec \frac{5\pi}{6} = \frac{1}{\cos \frac{5\pi}{6}} = \frac{1}{-\frac{\sqrt{3}}{2}} = -\frac{2}{\sqrt{3}}$

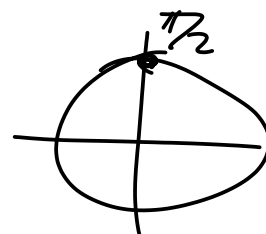
d) $\sin^{-1} \left(\sin \left(-\frac{3\pi}{4}\right)\right) = \sin^{-1} \left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$



e) $\sin^{-1} \left(\cos \frac{2\pi}{3}\right) = \sin^{-1} \left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

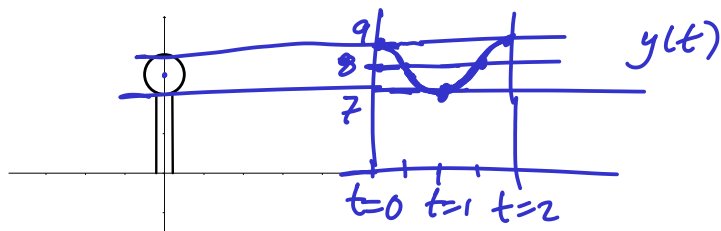
f) $\cos^{-1} \left(\sin \left(\tan^{-1}(0)\right)\right) = \cos^{-1}(\sin(0))$
 $= \cos^{-1}(0)$
 $= \frac{\pi}{2}$

$\tan 0 = 0 \Rightarrow$
 $\tan^{-1} 0 = 0$



2. You visit the abandoned Marlborough campus, and you see a mouse at the top of the clock tower. The mouse jumps onto the second hand of the clock and rides it around. The clock needs maintenance – it takes 2 minutes for the mouse to make a full revolution. You take the opportunity to practice your trigonometry and model the mouse's motion around the clock. You estimate that the clock's radius is 1 ft. and the bottom of the clock is 7 feet above the ground. Assume that the mouse is at the top of the clock at $t = 0$ minutes.

a) Graph both $x(t)$ and $y(t)$. Find equations for both $x(t)$ and $y(t)$.



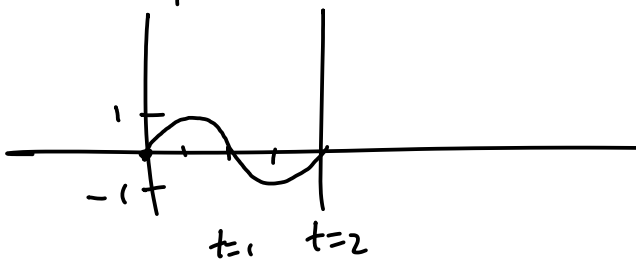
amplitude 1
period $\frac{2\pi}{b} = 2$

$$b = \pi$$

vertical shift 8

$$y(t) = \cos(\pi t) + 8$$

x -values (hand moves clockwise from top)



$$x(t) = \sin \pi t$$

b) Calculate the position $(x(t), y(t))$ of the mouse at $t = 3$, $t = 3.5$, and $t = 4$ minutes. Make sure your answers make sense.

$$t = 3: \quad x(3) = \sin 3\pi = 0$$

$$y(3) = (\cos 3\pi) + 8 = 7$$

$$(x, y) = (0, 7) \text{ bottom } \checkmark$$

(6)

$$t = \frac{7}{2}: \quad x\left(\frac{7}{2}\right) = \sin \frac{7\pi}{2} = -1$$

$$y\left(\frac{7}{2}\right) = 8 + \cos \frac{7\pi}{2} = 8$$

$$(x, y) = (-1, 8) \text{ left side } \checkmark$$

(9)

$$t = 4: \quad x(4) = \sin 4\pi = 0$$

$$y(4) = 8 + \cos(4\pi) = 9$$

$$(x, y) = (0, 9) \text{ top } \checkmark$$

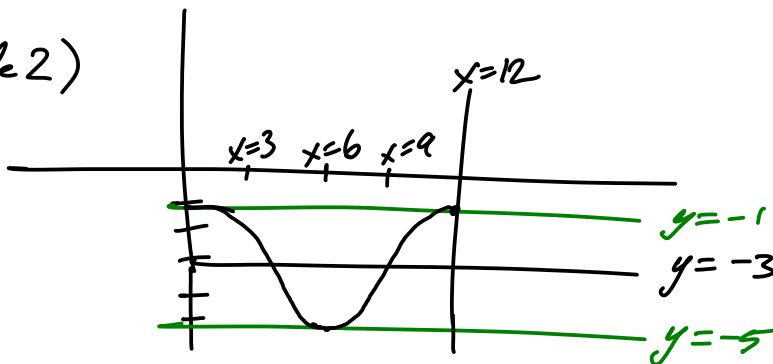
(12)

3. Write down all the relevant properties (period, amplitude, shifts/scales, asymptotes) of the following trig functions, and then graph by hand.

a. $f(x) = -3 + 2 \cos\left(\frac{\pi}{6}x\right)$ ← period $\frac{2\pi}{(\pi/6)} = 12$

vertical shift
-3

vertical scale
2
(amplitude 2)

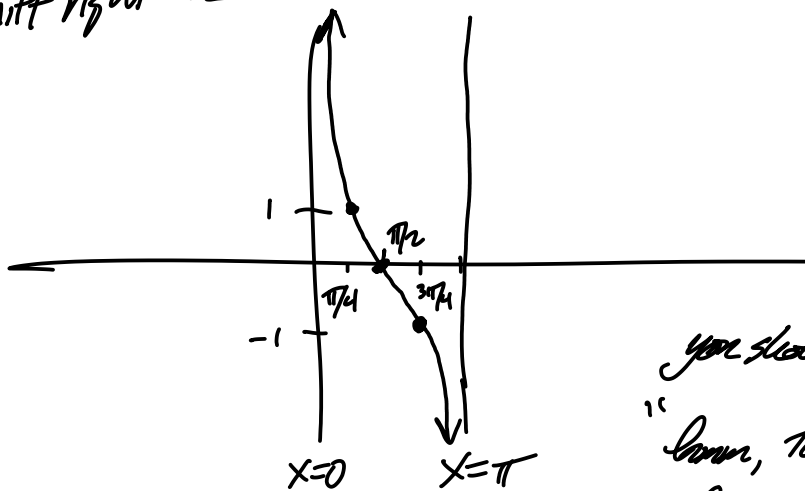


b. $g(x) = -\tan\left(x - \frac{\pi}{2}\right)$

vertical flip

shift right $\pi/2$

period π



you should be thinking:
"wow, this looks like cotangent..."