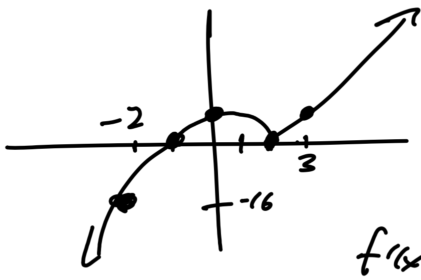
Unit 10 Test
PCHA 2020-21 / Dr. Kessner

Name/Pledge: KEY

No notes! No calculator! Have fun!

1. Graph the given function on the specified interval. Find all critical points. Identify any points where there is a local min/max, and verify with a derivative test. Identify the absolute max and min. If either fails to exist, state the condition of the Extreme Value Theorem that is *not* satisfied.

a. $f(x) = (x + 1)(x - 2)^2$ on $[-2, 3]$



$$f'(x) = 1 \cdot (x-2)^2 + (x+1)2(x-2)$$
$$= (x-2)[(x-2) + 2(x+1)]$$
$$= (x-2)[x-2+2x+2]$$
$$= 3x(x-2)$$

$$f(x) = (x+1)(x^2-4x+4)$$
$$= x^3 - 4x^2 + 4x + x^2 - 4x + 4$$
$$= x^3 - 3x^2 + 4$$
$$f'(x) = 3x^2 - 6x = 3x(x-2)$$

$f'(x) = 0 \Rightarrow x = 0,\ x = 2$

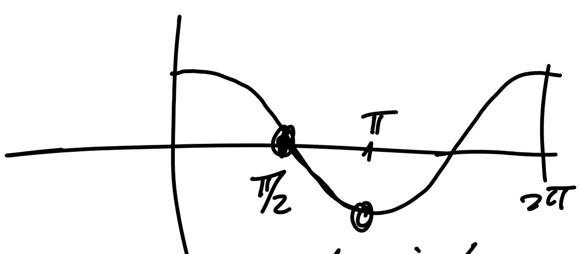
$f''(x) = 6x - 6 = 6(x-1)$

$f''(0) = -6$ local max

$f''(2) = 6$ local min

x	$f(x)$	
−2	−16	abs min
0	4	abs max
2	0	
3	4	abs max

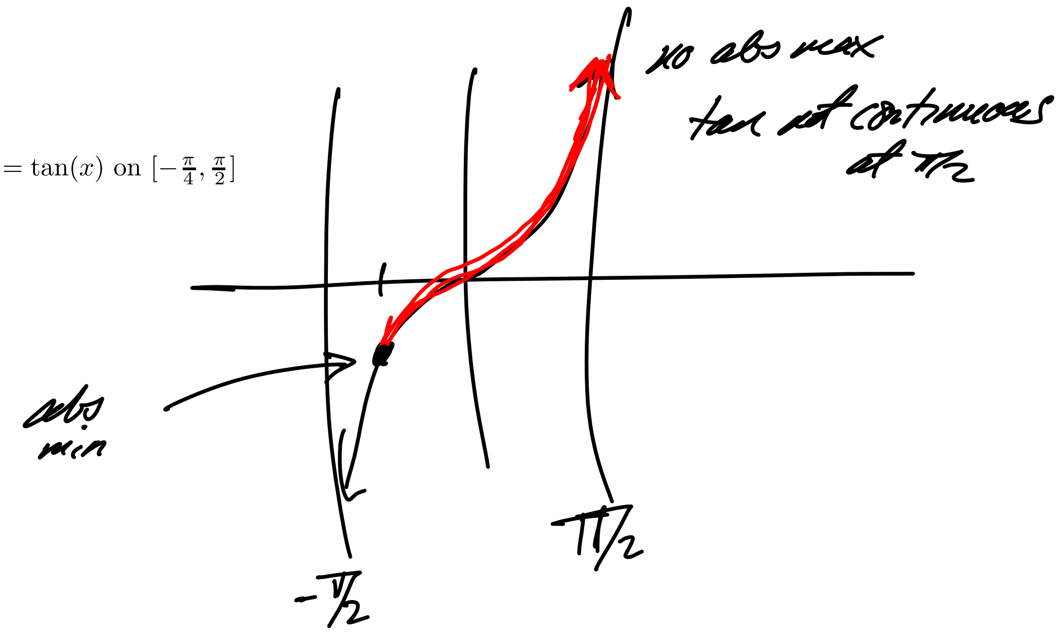
b. $g(x) = \cos x$ on $(\frac{\pi}{2}, \pi)$



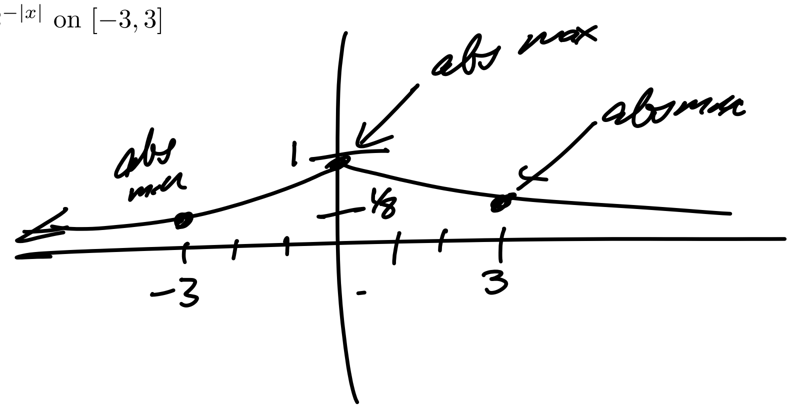
no abs min/max

$(\frac{\pi}{2}, \pi)$ not closed

c. $h(x) = \tan(x)$ on $[-\frac{\pi}{4}, \frac{\pi}{2}]$



d. $j(x) = 2^{-|x|}$ on $[-3, 3]$



2. For each of the given functions find all antiderivatives.

a. $k'(x) = -\cos 5x$

$$k(x) = -\frac{1}{5}\sin 5x + C$$

b. $l'(x) = 7x^3 + x^2$

$$l(x) = \frac{7}{4}x^4 + \frac{x^3}{3} + C$$

c. $m'(x) = x^{-3} + x^{-1} + x^1$

$$m(x) = \frac{x^{-2}}{-2} + \ln(x) + \frac{x^2}{2} + C$$

d. $n'(x) = 7^x + x^7$

$$n(x) = \frac{1}{\ln 7}7^x + \frac{x^8}{8} + C$$

e. $p'(x) = e^{5x+1}$

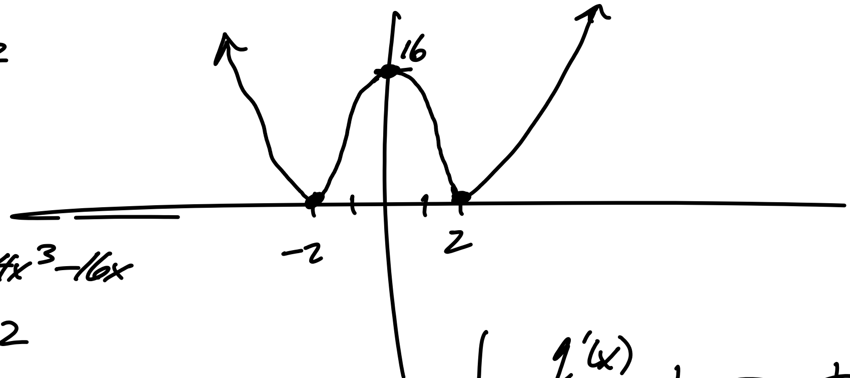
$$p(x) = \frac{e^{5x+1}}{5} + C$$

3. For each of the following functions: sketch the graph, then find all local extrema and verify with a derivative test.

a. $q(x) = x^4 - 8x^2 + 16$ *(Hint: factor as a perfect square)*

$= (x^2-4)^2$

$= (x+2)^2(x-2)^2$



$q'(x) = 2(x^2-4)(2x)$

$= 4x(x^2-4) = 4x^3 - 16x$

$q'(x) = 0 \Rightarrow x = 0, \pm 2$

$q''(x) = 12x^2 - 16$

$= 4(x^2-4)$

$q''(0) = -16$ local max

$q''(\pm 2) = 0 \Rightarrow ?$

$q'(x)$: $-$ | $+$ | $-$ | $+$ at -2, 0, 2

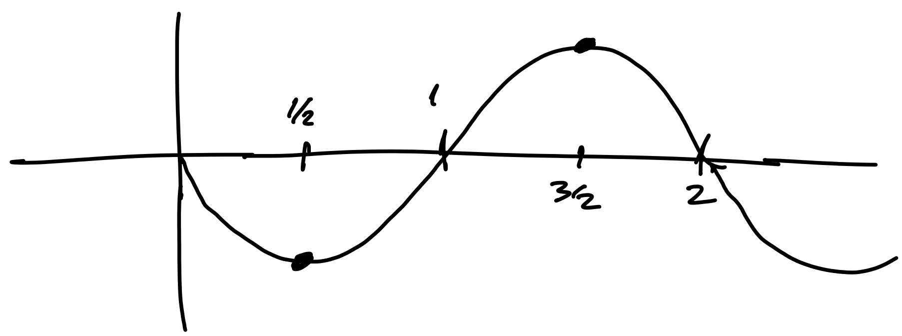
1st deriv test

$\Rightarrow x = \pm 2$ local min

or argue $q(x) \geq 0$

b. $r(x) = -3\sin \pi x$ period 2

$r'(x) = -3\pi \cos \pi x$



$r'(x) = 0 \Rightarrow \cos \pi x = 0$

$\pi x = \frac{\pi}{2} + k\pi$

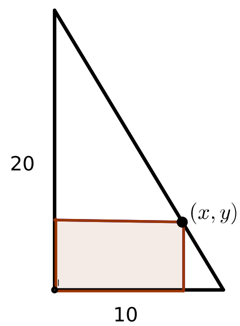
$x = \frac{1}{2} + k$

$r''(x) = +3\pi^2 \cos \pi x$

$r''(\frac{1}{2} + 2k) = 3\pi^2 > 0$ local min

$r''(\frac{3}{2} + 2k) = -3\pi^2 < 0$ local max

4.



You are installing a rectangular robot playing field in a triangular room, and you want to maximize the area of the playing field.

a. Write your constraint on x and y (the equation of the diagonal line).

$$y = 20 - 2x$$

b. Write an equation for the function you want to maximize (the area of the rectangle). Use your constraint to eliminate one of the variables.

$$A = xy = x(20 - 2x) = 20x - 2x^2$$
$$= 2x(10 - x)$$

c. Maximize your function by taking the derivative and finding any critical points. Be sure to use a derivative test to verify that you have found a local max. *Bonus: How do we know that this is actually an absolute maximum?*

$$A'(x) = 20 - 4x$$
$$A'(x) = 0 \Rightarrow x = 5 \quad y = 10$$
$$A''(x) = -4 \quad \text{local max}$$