Group Work Unit 2
PCHA 2021-22 / Dr. Kessner

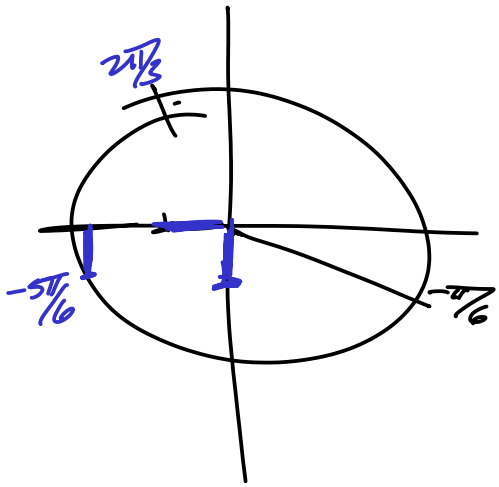
Name: KEY

Partner(s):

You can use any materials from class, but no other online resources.
No calculator except where indicated. Have fun!

1. Evaluate the following:

a) $\sec \frac{2\pi}{3}$ $= -2$

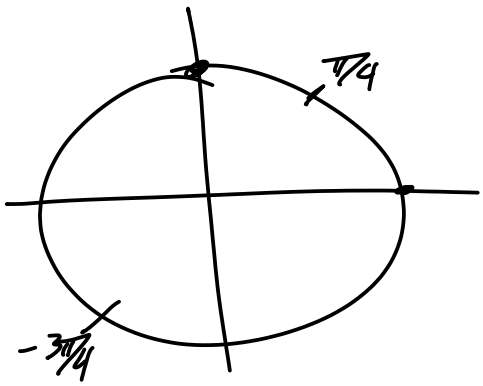


b) $\csc(-\frac{5\pi}{6})$ $= -2$

c) $\cos^{-1}\left(\underbrace{\cos \frac{2\pi}{3}}_{-1/2}\right) = \frac{2\pi}{3}$

d) $\sin^{-1}\left(\underbrace{\sin(-\frac{5\pi}{6})}_{-1/2}\right) = -\frac{\pi}{6}$

e) $\tan^{-1}\left(\underbrace{\cot(-\frac{3\pi}{4})}_{1}\right) = \pi/4$



f) $\tan\left(\sin^{-1}\underbrace{(\cos \frac{\pi}{2})}_{0}\right) = \underbrace{0}_{0} = 0$

2. Write down all the relevant properties (period, amplitude, shifts/scales, asymptotes) of the following trig functions, and then graph by hand.

$f(x) = 15 - 10\sin(\frac{\pi}{8}x - \frac{\pi}{2})$

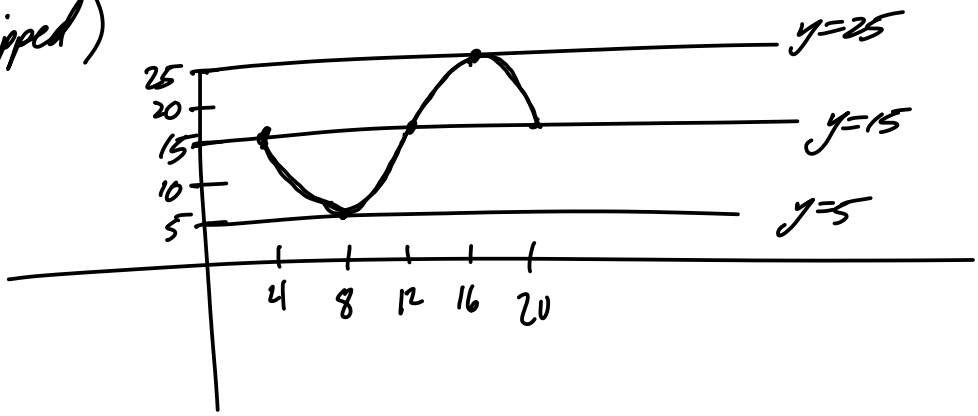
$= 15 - 10\sin[\frac{\pi}{8}(x-4)]$

vertical shift 15

amplitude 10 (flipped)

shift right 4

period $\frac{2\pi}{\pi/8} = 16$

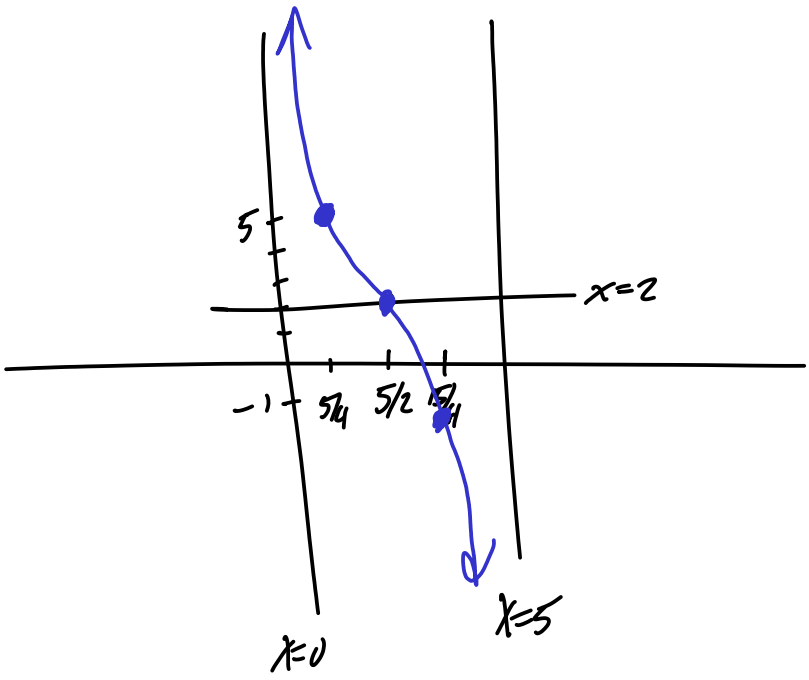


$g(x) = 3\cot(\frac{\pi}{5}x) + 2$

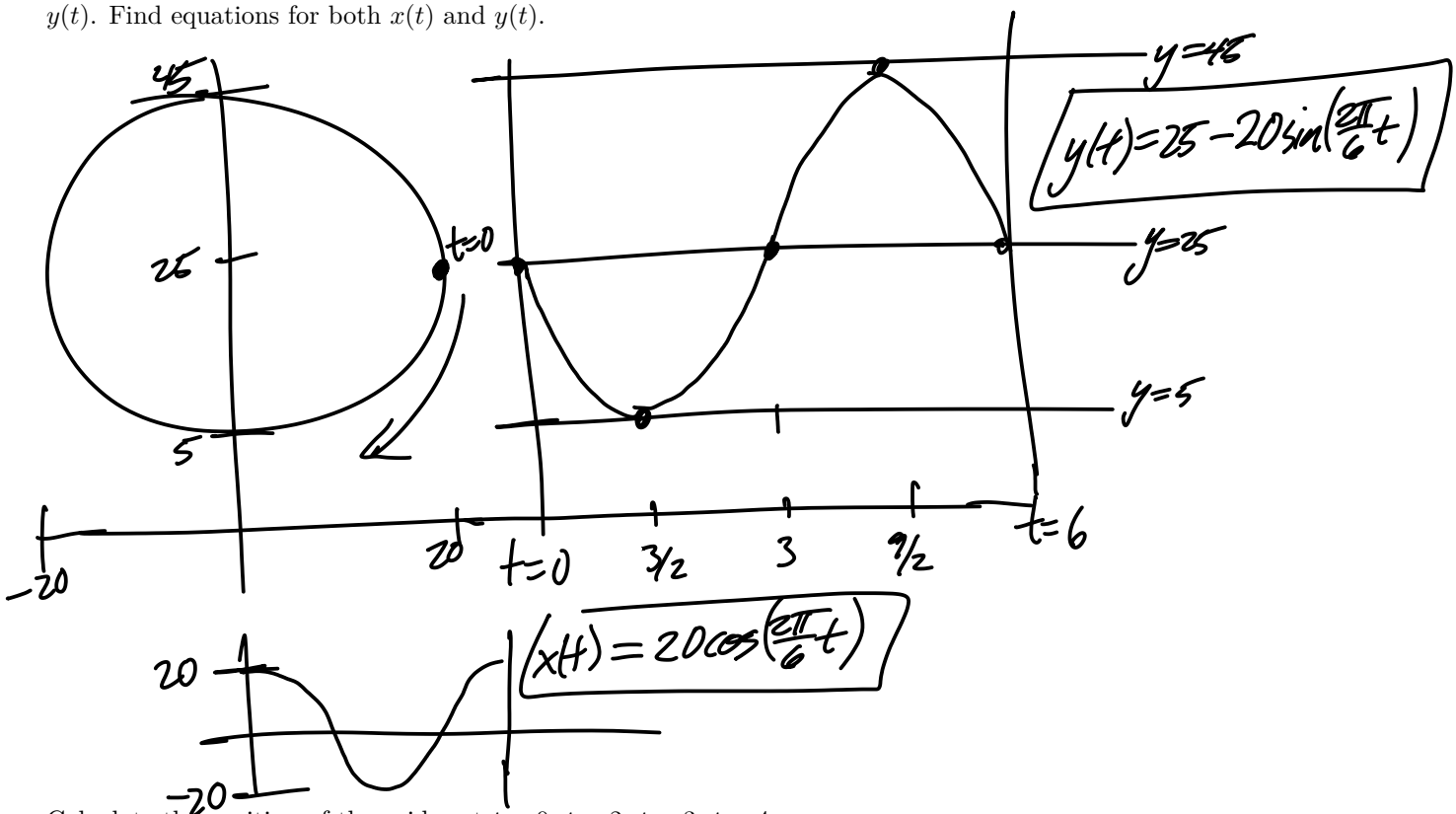
vertical scale x3

vertical shift 2

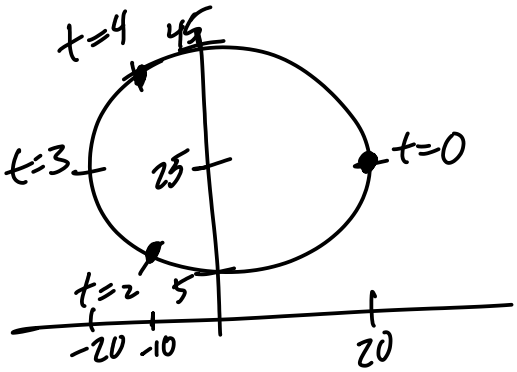
period $\frac{\pi}{(\pi/5)} = 5$



3. You model the motion of a spider sitting on the wheel of a stationary bike. The spider starts ($t = 0$) at the 3 o'clock position and travels clockwise with a constant angular speed, taking 6 seconds for a full revolution. The wheel's lowest point is 5cm above the ground, and it's radius is 20cm. Graph both $x(t)$ and $y(t)$. Find equations for both $x(t)$ and $y(t)$.



Calculate the position of the spider at $t = 0$, $t = 2$, $t = 3$, $t = 4$.



$t=0$ $x(0)=20\cos 0=20$
$y(0)=25-20\sin(0)=25$

$t=2$ $x(2)=20\cos\frac{2\pi}{3}=-10$
$y(2)=25-20\sin\frac{2\pi}{3}=25-\frac{20\sqrt{3}}{2}=25-10\sqrt{3}$

$t=3$ $x(3)=20\cos\pi=-20$
$y(3)=25-20\sin\pi=25$

$t=4$ $x(4)=20\cos\frac{4\pi}{3}=-10$
$y(4)=25-20\sin\frac{4\pi}{3}=25+10\sqrt{3}$

4. Prove the identities:

$\dfrac{1}{\csc^2 x(1-\cos x)} = 1+\cos x$

$\dfrac{1}{\csc^2 x(1-\cos x)} = \dfrac{\sin^2 x}{1-\cos x}$

$= \dfrac{1-\cos^2 x}{1-\cos x}$

$= \dfrac{(1-\cos x)(1+\cos x)}{1-\cos x}$

$= 1+\cos x$ ✓

this derivation from Katie

or do: $\dfrac{1}{\csc^2 x(1-\cos x)} = \dfrac{1}{\csc^2 x(1-\cos x)} \cdot \dfrac{1+\cos x}{1+\cos x}$

...

$(\cot x + \tan x)^2 = \sec^2 x + \csc^2 x$

$(\cot x+\tan x)^2 = \cot^2 x + 2\cdot \underbrace{\cot x \tan x}_{1} + \tan^2 x$

$= (\cot^2 x + 1) + (1+\tan^2 x)$

$= \csc^2 x + \sec^2 x$ ✓

5. Find $\sin(75°)$ using a sum angle formula.

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$\sin 75° = \sin(30° + 45°)$$
$$= \sin 30° \cos 45° + \cos 30° \sin 45°$$
$$= \frac{1}{2}\cdot\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$
$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$

Derive the following half angle formula from the relevant double angle formula:

$$\sin u = \pm\sqrt{\frac{1-\cos 2u}{2}}$$

$$\cos 2u = \cos^2 u - \sin^2 u$$
$$\cos 2u = 1 - 2\sin^2 u$$
$$2\sin^2 u = 1 - \cos 2u$$
$$\sin^2 u = \frac{1-\cos 2u}{2}$$
$$\sin u = \pm\sqrt{\frac{1-\cos 2u}{2}}$$

Use the half angle formula above to find $\sin(75°)$.

$$\sin 75° = +\sqrt{\frac{1-\cos 150°}{2}} = \sqrt{\frac{1+\sqrt{3}/2}{2}}$$

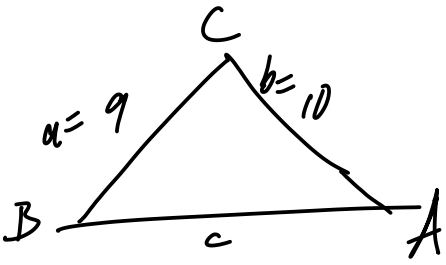
quadrant 1

$$= \frac{1}{2}\sqrt{2+\sqrt{3}}$$

Challenge: Show that the two expressions you found for $\sin(75°)$ are equal.

6. Solve the following triangle. **Calculator OK**

a = 9, b = 10, A = 60°



$\frac{\sin B}{b} = \frac{\sin A}{a}$

$\sin B = \frac{b \sin A}{a}$

$= \frac{10 \sin 60°}{9}$

$= \frac{10}{9} \frac{\sqrt{3}}{2}$

$\approx .96225$

$B_2 \approx$

$B_1 \approx 74.2°$ or $105.8°$

$C_1 \approx 45.8°$ $\quad C_2 \approx 14.2°$

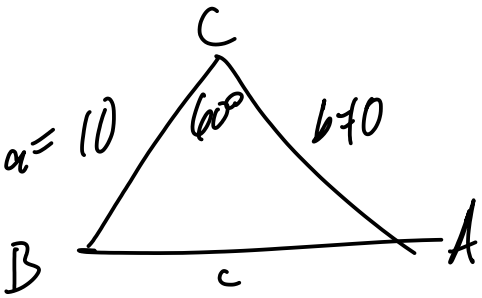
$c_1^2 \approx a^2 + b^2 - 2ab\cos C_1$ $\quad c_2^2 \approx 6.5$

≈ 55.5 $\quad c_2 \approx 2.5$

$c_1 \approx 7.45$

Solve the following triangle. **No calculator!**

a = 10, b = 10, C = 60°



$c^2 = a^2 + b^2 - 2ab\cos C$

$= 10^2 + 10^2 - 2 \cdot 10 \cdot 10 \cdot \cos 60°$

$= 100 + 100 - 200\left(\frac{1}{2}\right)$

$= 100$

$c = 10$

$\frac{\sin A}{a} = \frac{\sin C}{c} \Rightarrow \sin A = \frac{a \sin C}{c}$

$= \frac{10 \cdot \sin 60°}{10}$

$\Rightarrow A = 60°$

(similar: $B = 60°$)