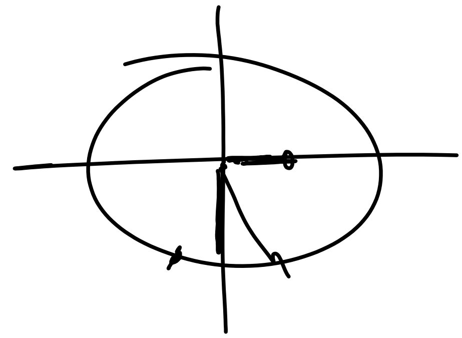
Test Unit 2
PCHA 2020-21 / Dr. Kessner

Name:

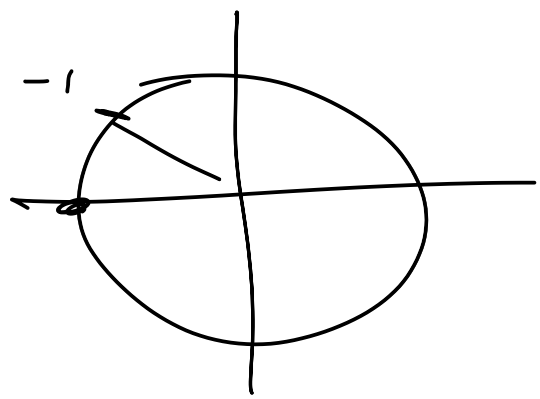
No calculator, no notes – just your brain! Have fun!

1. Evaluate the following:

a) $\sec\frac{5\pi}{3} = \frac{1}{(1/2)} = 2$

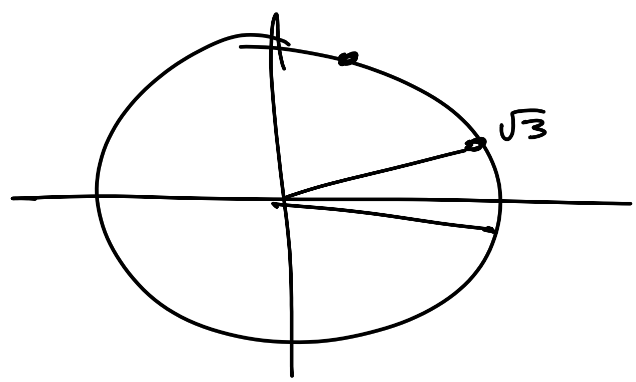


b) $\sin(-\frac{2\pi}{3}) = -\frac{\sqrt{3}}{2}$



c) $\cos^{-1}\left(\underbrace{\cot(-\frac{5\pi}{4})}_{-1}\right) = \pi$

d) $\sin^{-1}\left(\underbrace{\cos(-\frac{\pi}{2})}_{0}\right) = 0$



e) $\tan^{-1}\left(\underset{\sqrt{3}}{\cot(\frac{\pi}{6})}\right) = \frac{\pi}{3}$

f) $\sin(-\frac{\pi}{12}) = \sin\left(\frac{\pi}{4} - \frac{\pi}{3}\right)$

$= \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{2}\sin\left(\frac{-\pi}{3}\right)$

$= \frac{\sqrt{2}}{2}\frac{1}{2} + \frac{\sqrt{2}}{2}\left(-\frac{\sqrt{3}}{2}\right)$

$= \frac{\sqrt{2}-\sqrt{6}}{4}$

2. Write down all the relevant properties (period, amplitude, shifts/scales, asymptotes) of the following trig functions, and then graph by hand.

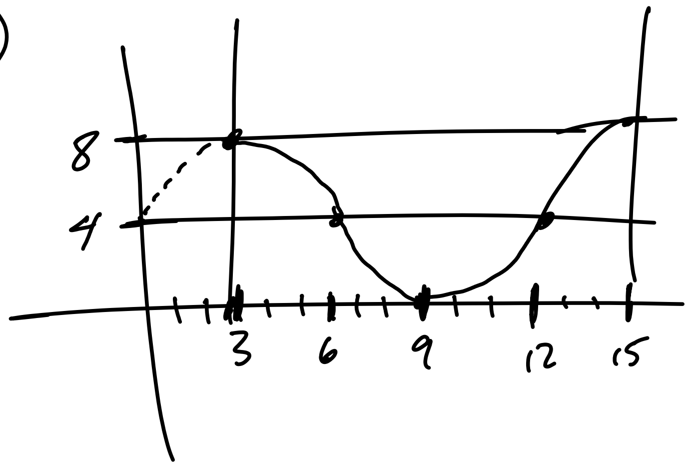
$f(x) = 4 + 4\cos\left(\frac{\pi}{6}x - \frac{\pi}{2}\right)$

$= 4 + 4\cos\left(\frac{\pi}{6}(x-3)\right)$

period $\frac{2\pi}{(\pi/6)} = 12$

shift right 3, up 4

amplitude 4



$g(x) = 5\cot\left(\frac{\pi}{8}(x-4)\right)$

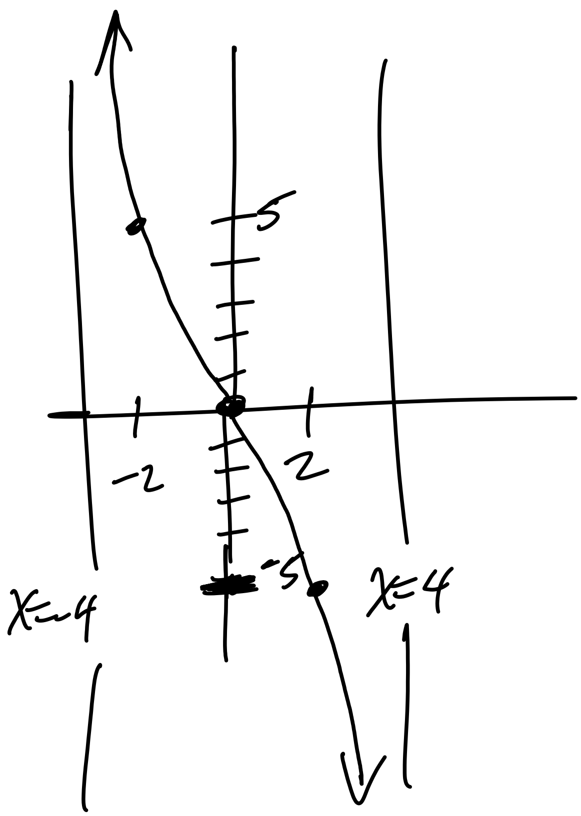
period $\frac{\pi}{(\pi/8)} = 8$

shift left 4

right

but no change in graph

vertical scale 5



Bonus Write f as a transformed sin and g as a transformed tan.

$4 + 4\sin\left(\frac{\pi}{6}x\right)$ $\quad -5\tan\left(\frac{\pi}{8}x\right)$

3. Prove the identities:

$$\frac{\cos x + \sin x}{\cos x - \sin x} = \sec 2x + \tan 2x$$

$$\frac{\cos x + \sin x}{\cos x - \sin x} \cdot \frac{\cos x + \sin x}{\cos x + \sin x}$$

$$= \frac{(\cos x + \sin x)^2}{\cos^2 x - \sin^2 x}$$

$$= \frac{\cos^2 x + \sin^2 x + 2\sin x \cos x}{\cos^2 x - \sin^2 x}$$

$$= \frac{1 + \sin 2x}{\cos 2x}$$

$$= \sec 2x + \tan 2x \checkmark$$

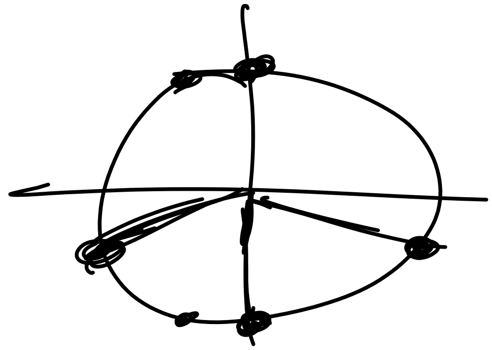
$$sin(\pi - x) = \sin x$$

$$\sin(\pi - x) = \underbrace{\sin \pi}_{0} \cos x - \underbrace{\cos \pi}_{-1} \sin x$$

$$= \sin x \checkmark$$

Bonus Prove this using cofactor identities.

4. Find all solutions of $\sin 2\theta + \cos\theta = 0$.



$$2\sin\theta\cos\theta + \cos\theta = 0$$

$$\cos\theta(2\sin\theta + 1) = 0$$

$$\cos\theta = 0 \quad \text{or} \quad \sin\theta = -\frac{1}{2}$$

$$\theta = \frac{\pi}{2} + \pi k \qquad \theta = -\frac{\pi}{6} + 2\pi k$$

$$-\frac{5\pi}{6} + 2\pi k$$

Derive the following half angle formula from the relevant double angle formula:

$$\cos u = \pm\sqrt{\frac{1+\cos 2u}{2}}$$

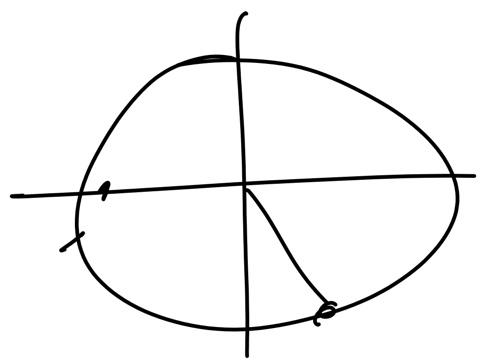
$$\cos 2u = \cos^2 u - \sin^2 u$$

$$= 2\cos^2 u - 1$$

$$2\cos^2 u = 1 + \cos 2u$$

$$\cos^2 u = \frac{1+\cos 2u}{2}$$

$$\cos u = \pm\sqrt{\frac{1+\cos 2u}{2}}$$



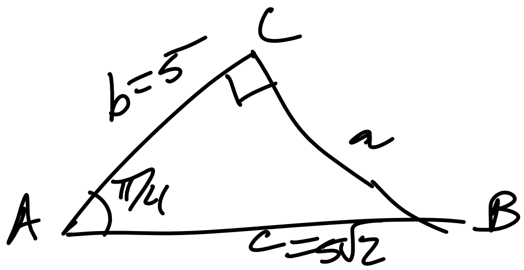
Use the half angle formula above to find $\cos(-\frac{5\pi}{12})$.

$$\cos\left(-\frac{5}{12}\pi\right) = +\sqrt{\frac{1+\cos\left(-\frac{5\pi}{6}\right)}{2}}$$

$$= \sqrt{\frac{1+\left(-\sqrt{3}/2\right)}{2}}$$

$$= \frac{1}{2}\sqrt{2-\sqrt{3}}$$

5. Solve the following triangle: $A = \frac{\pi}{4}$, $b = 5$, $c = 5\sqrt{2}$.



$$a^2 = b^2 + c^2 - 2abc\cos A$$

$$= 25 + 50 - 2 \cdot 5 \cdot 5\sqrt{2}\,\underbrace{\cos\frac{\pi}{4}}_{\sqrt{2}/2}$$

$$= 25 + 50 - 50$$

$$= 25$$

$$a = 5$$

$$\sin C = \frac{c \sin A}{a} = \frac{5\sqrt{2}\sin \pi/4}{5} = 1$$

$$\Rightarrow C = \pi/2$$

$$B = \pi/4$$