

Unit 4 Group Work 2
PCHA 2021-22 / Dr. Kessner

No calculator! Have fun!

1. Let

$$f(x) = \begin{cases} x + 1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ \cos x & \text{if } x > 0 \end{cases}$$

a) Sketch the graph of $f(x)$.

b) On what intervals is f increasing and/or decreasing? Is f bounded? Does it have any local or global maxima or minima?

c) Does f have any discontinuities? Where, and what type?

d) Describe the end behavior of f using limits.

2. Consider the same function from the previous problem.

$$f(x) = \begin{cases} x + 1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ \cos x & \text{if } x > 0 \end{cases}$$

Sketch the graphs of the following transformed functions:

- $p(x) = -f(x)$
- $q(x) = f(|x|)$
- $r(x) = -f(|x|)$
- $s(x) = f(-|x|)$
- $t(x) = |f(-|x|)|$

3. Factor the following polynomial completely, both over \mathbb{R} (as a product of real linear and irreducible quadratic factors) and over \mathbb{C} (as a product of complex linear factors). Sketch the graph of the function.

$$p(x) = x^4 + 6x^3 + 13x^2 + 12x + 4$$

4. Factor the following polynomial completely, both over \mathbb{R} (as a product of real linear and irreducible quadratic factors) and over \mathbb{C} (as a product of complex linear factors). Sketch the graph of the function.

$$q(x) = x^5 + 2x^4 - 16x - 32$$

5. Sketch the graph of the following rational function.

$$r(x) = \frac{x^3 + x^2 - x - 1}{x}$$

Write limits to describe its end behavior, and its behavior near asymptotes. Challenge: Describe its asymptotic end behavior.