Unit 4 Group Work Practice
PCHA 2021-22 / Dr. Kessner

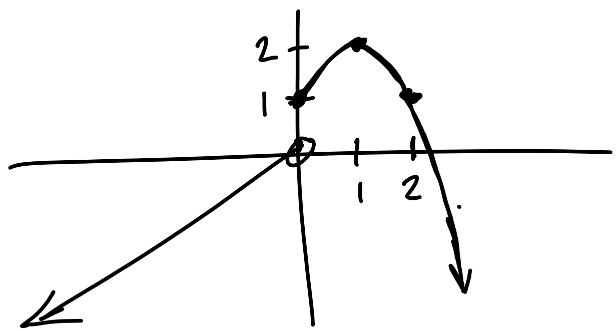
KEY

No calculator! Have fun!

1. Let

$$f(x) = \begin{cases} x & \text{if } x < 0 \\ 2-(x-1)^2 & \text{if } x \geq 0 \end{cases}$$

a) Sketch the graph of $f(x)$.



b) On what intervals is f increasing and/or decreasing? Is f bounded? Does it have any local or global maxima or minima?

increasing on $(-\infty, 1]$
decreasing on $[1, \infty)$
bounded above
local & global max at $x=1$

c) Does f have any discontinuities? Where, and what type?

jump discontinuity at $x=0$

d) Describe the end behavior of f using limits.

$\lim_{x \to \infty} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = -\infty$

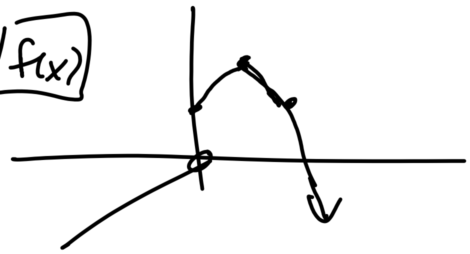
2. Consider the same function from the previous problem.

$$f(x) = \begin{cases} x & \text{if } x < 0 \\ -(x-1)^2 & \text{if } x \geq 0 \end{cases}$$

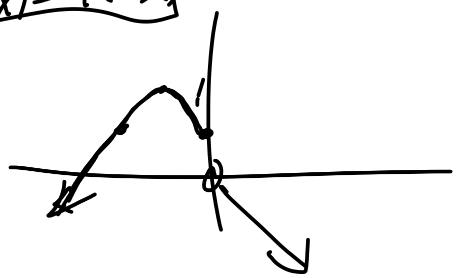
Sketch the graphs of the following transformed functions:

- $p(x) = f(-x)$
- $q(x) = f(|x|)$
- $r(x) = |f(|x|)|$
- $s(x) = |f(-x)|$
- $t(x) = -f(-|x|)$

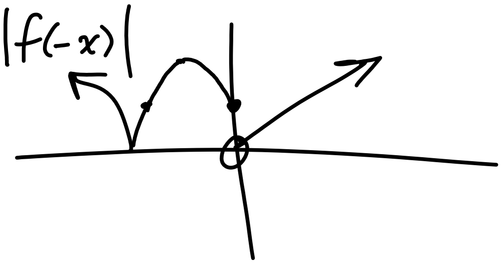
$\boxed{f(x)}$



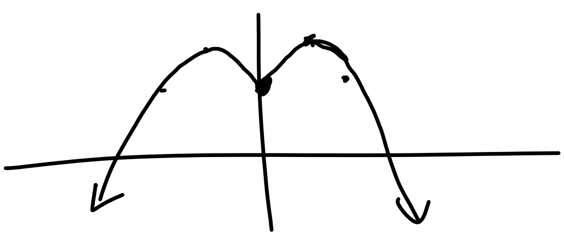
$\boxed{p(x) = f(-x)}$



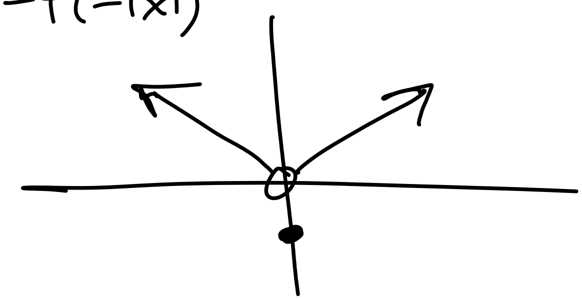
$s(x) = |f(-x)|$



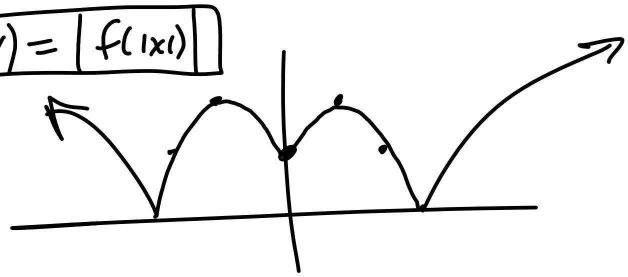
$\boxed{q(x) = f(|x|)}$



$t(x) = -f(-|x|)$



$\boxed{r(x) = |f(|x|)|}$



3. Factor the following polynomial completely, both over $\mathbb{R}$ (as a product of real linear and irreducible quadratic factors) and over $\mathbb{C}$ (as a product of complex linear factors). Sketch the graph of the function.

$$p(x) = -2x^3 + 7x^2 + 17x - 10$$

potential rational zeros: $\pm \frac{1, 2, 5, 10}{1, 2}$

$p(1) = -2 + 7 + 17 - 10 \neq 0$
$p(-1) = +2 + 7 - 17 - 10 \neq 0$
$p(2) = -16 + 28 + 34 - 10 \neq 0$
$p(-2) = 16 + 28 - 34 - 10 = 0$ ✓

$$\begin{array}{r|rrrr} \underline{-2} & -2 & 7 & 17 & -10 \\ & & 4 & -22 & 10 \\ \hline & -2 & 11 & -5 & \boxed{0} \end{array}$$

$p(x) = (x+2)(-2x^2 + 11x - 5)$

zeros at $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-11 \pm \sqrt{121 - 40}}{-4}$

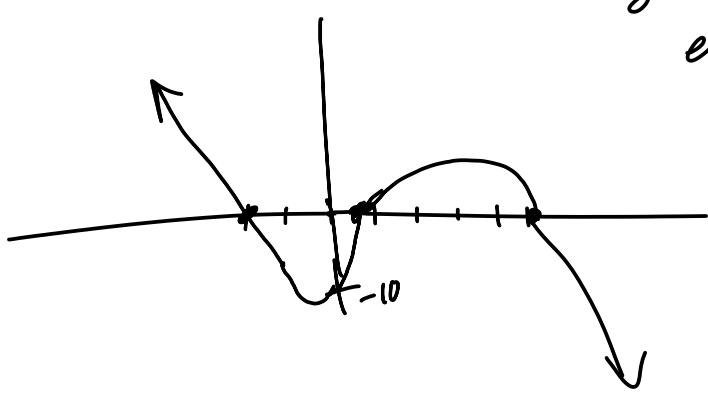
$= \frac{-11 \pm 9}{-4}$

$= 5, \frac{1}{2}$

or factor $-(2x^2 - 11x + 5)$
$-(2x-1)(x-5)$

$\Rightarrow p(x) = -(x+2)(2x-1)(x-5)$

zeros: $-2, \frac{1}{2}, 5$
end behavior:
$\lim_{x \to \infty} p(x) = -\infty$
$\lim_{x \to -\infty} p(x) = +\infty$
y-intercept -10



4. Factor the following polynomial completely, both over $\mathbb{R}$ (as a product of real linear and irreducible quadratic factors) and over $\mathbb{C}$ (as a product of complex linear factors). Sketch the graph of the function. A little bird tells you that $2+3i$ is a zero.

$$q(x) = x^4 - 4x^3 + 10x^2 + 12x - 39$$

$$\begin{array}{r|rrrrr}
2+3i & 1 & -4 & 10 & 12 & -39 \\
 & & 2+3i & -13 & -6-9i & 39 \\
\hline
2-3i & 1 & -2+3i & -3 & 6-9i & \boxed{0} \\
 & & 2-3i & 0 & -6+9i & \\
\hline
 & 1 & 0 & -3 & \boxed{0} &
\end{array}$$

$(2+3i)(-2+3i)$
$= -4-9 = -13$

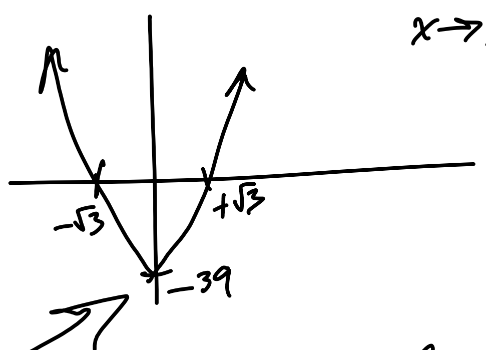
$(2+3i)(6-9i)$
$= 12+27 = 39$

$$\begin{aligned}
\Longrightarrow q(x) &= (x-(2+3i))(x-(2-3i))(x^2-3) \\
&= (x-(2+3i))(x-(2-3i))(x+\sqrt{3})(x-\sqrt{3}) \quad \text{over } \mathbb{C} \\
&= (x^2-4x+13)\underbrace{(x+\sqrt{3})(x-\sqrt{3})}_{\text{2 real (irrational) zeros}} \quad \text{over } \mathbb{R}
\end{aligned}$$

end behavior:

$$\lim_{x\to\pm\infty} q(x) = \infty$$

y-intercept -39



probably not local min

5. Sketch the graph of the following rational function:

$$r(x) = \frac{x^2 - 3x + 2}{(x^2 - 4x + 4)(x - 3)}$$

$$= \frac{(x-2)(x-1)}{(x-2)^2(x-3)}$$

$$= \frac{x-1}{(x-2)(x-3)} \quad (\text{if } x \neq 2)$$

end behavior: $\lim_{x \to \pm\infty} r(x) = 0$

		1		2		3	
$x-1$	−	0	+	+	+	+	+
$x-2$	−	−	−	0	+	+	+
$x-3$	−	−	−	−	−	0	+
$r(x)$	−	0	+	X	−	X	+