Unit 6 Group Work
PCHA 2021-22 / Dr. Kessner

No Calculator

1. Evaluate:

a. $\binom{7}{1}$

b. $\binom{7}{2}$

c. $\binom{7}{3}$

d. $\binom{7}{4}$

e. $\binom{12}{2}$

f. $\binom{12}{3}$

g. $\binom{12}{9}$

h. $\binom{12}{10}$

i. $\binom{100}{99}$

j. $\binom{2000}{2}$

2. Let $\{a_k\}_{k=1}^{\infty} = \{\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \cdots\}$.

a. What type of sequence is this? Write recursive and explicit formulas for a_k.

b. Let S_n be the n^{th} partial sum of the sequence $\{a_k\}$. Express S_n (for this particular sequence) in summation notation.

c. Write a formula for the actual sum S_n (for this particular sequence).

d. What is the sum of the infinite series $\frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \cdots$? (Surprising?)

3. Expand $(2 - x^2)^4$.

Find the x^6 term in $(2 - x^2)^5$.

Find the x^8 term in $(2 - x^2)^5$.

4. Suppose you have 7 red and 3 white marbles in a bag. You pick 6 of the marbles from the bag (without replacement).

a. What is the probability that you pick 6 red marbles?

b. What is the probability that you pick 4 red (and 2 white marbles)?

c. What is the probability that you pick 2 red marbles?

5. Suppose you have 50 black and 50 white marbles in a bag. You sample 5 marbles with replacement (in other words, you pick a marble, look at it, and put it back, 5 times). Let B be the number of times you pick a black marble. Calculate all of the 6 probabilities $P(B = 0), P(B = 1), ..., P(B = 5)$. *Hint:* You only have to calculate half of these. Verify that $1 = \sum_{k=0}^{5} P(B = k)$.

Now suppose you have 75 black and 25 white marbles in the bag. You again sample 5 marbles with replacement. Calculate the probabilities $P(B = 0), P(B = 1), ..., P(B = 5)$ and again verify that $1 = \sum_{k=0}^{5} P(B = k)$.