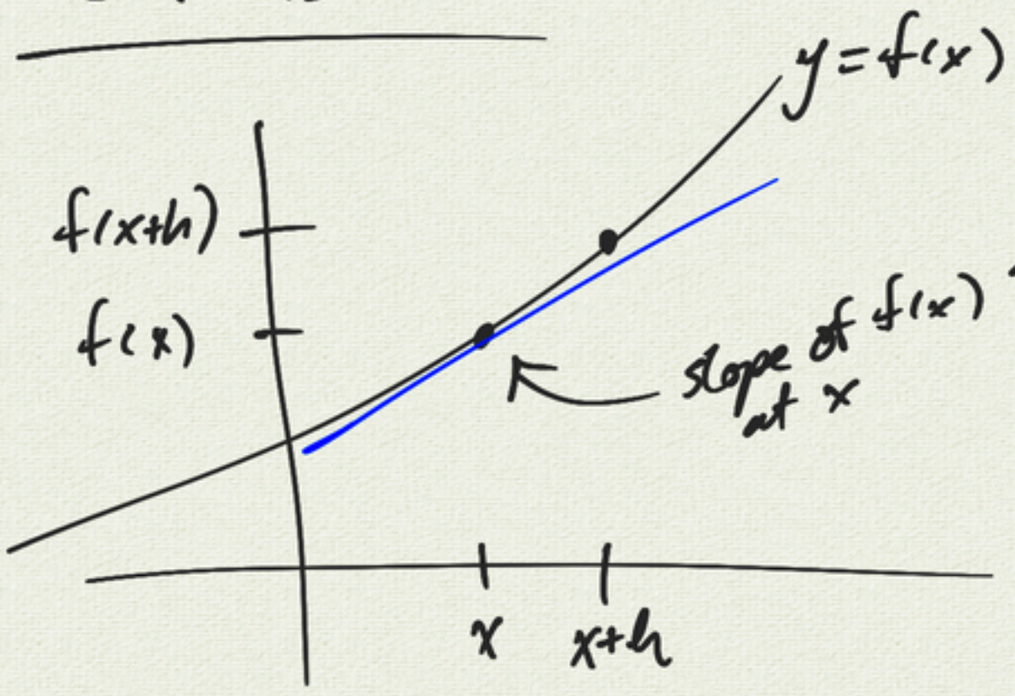
Calculus



$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

instaneous rate of change

slope of tangent line

rules:

$\frac{d}{dx}(\text{const}) = 0$

$\frac{d}{dx}(x^n) = nx^{n-1}$ power rule

$\frac{d}{dx}(kf) = k\frac{df}{dx}$

$\frac{d}{dx}(f+g) = \frac{df}{dx} + \frac{dg}{dx}$ "linearity"

$\frac{d}{dx}(fg) = f'g + fg'$ product rule

$\frac{d}{dx}(f \circ g) = f'(g(x)) \cdot g'(x)$ chain rule

example:

$\frac{d}{dx}(x^3 + 5x^2 + 3)$

$= 3x^2 + 10x$

quotient rule:

$\frac{d}{dx}(f/g) = \frac{d}{dx}(fg^{-1})$

$= f'(g)^{-1} + f(-1)(g)^{-2} \cdot g'$

$= \frac{f'g - fg'}{g^2}$

$\frac{d}{dx}(\sin x) = \cos x$ $\quad$ $\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$ $\quad$ $\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\sec x) = \sec x \tan x$ $\quad$ $\frac{d}{dx}(\csc x) = -\csc x \cot x$

$\frac{d}{dx}(e^x) = e^x$ $\quad$ $\frac{d}{dx}(\ln x) = \frac{1}{x}$

$\frac{d}{dx}(a^x) = a^x \ln a$ $\quad$ $\frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$

---

implicit diff: $y = \ln x$

$\Rightarrow e^y = x$

$e^y \frac{dy}{dx} = 1 \cdot \frac{dx}{dx}$

$\frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$

$\frac{d}{dx}(\ln x) = \frac{1}{x}$

applications:

1. projectile motion
2. Ferris wheel

] parametric equations

3. exponential growth/decay

polynomials, trig, exp/log

---

special limits:

$\lim_{x \to 0} \frac{\sin x}{x} = 1$

$\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$

(1a) $\lim_{x\to 0} x\cot 7x = \lim_{x\to 0} \frac{x\cos 7x}{\sin 7x}\cdot\frac{7}{7}$

$= \lim_{x\to 0}\left(\frac{7x}{\sin 7x}\right)(\cos 7x)\cdot\frac{1}{7}$

$= \frac{1}{7}$

$\lim_{x\to 0}\frac{\sin x}{x} = 1$

$\lim_{x\to 0}\frac{\sin 7x}{7x} = 1$

$\lim_{x\to 0}\frac{7x}{\sin 7x} = 1$

(5a) 4000 cells, 4 hour doubling time

t	$P(t)$
0	$4000 = 4000$
4	$8000 = 4000\cdot 2^{1}$
8	$16000 = 4000\cdot 2^{2}$
12	$32000 = 4000\cdot 2^{3}$

$P(t) = 4000\cdot 2^{t/4}$

#doubling times

(3e) $t(x) = \log_2(\sec^3(x^5))$

$t'(x) = \frac{1}{\sec^3(x^5)\ln 2}\cdot 3(\sec(x^5))^2\cdot\sec(x^5)\tan(x^5)\cdot 5x^4$

(3d) $s(x) = e^{\cot(x^3-1)}$ $(f\circ g)(x)$ $f(x)=e^x$, $g(x)=\cot(x^3-1)$

$s'(x) = \underbrace{e^{\cot(x^3-1)}}_{f'(g(x))}\cdot\underbrace{(-\csc^2(x^3-1))\cdot(3x^2)}_{g'(x)}$

(6c) $x(t) = 6\cos\left(\frac{2\pi}{3}t\right)$

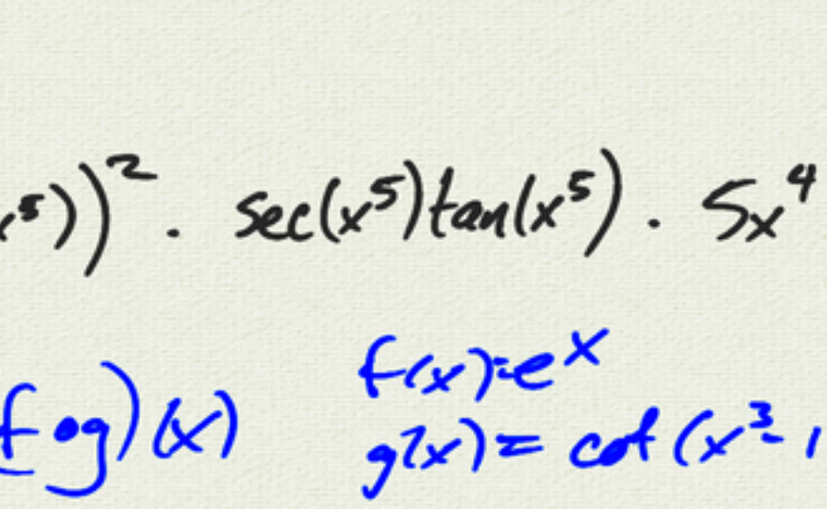
$y(t) = 6\sin\left(\frac{2\pi}{3}t\right) + 16$

$x'(t) = 6\left(-\sin\left(\frac{2\pi}{3}t\right)\right)\left(\frac{2\pi}{3}\right)$

$y'(t) = \left(6\cos\frac{2\pi}{3}t\right)\left(\frac{2\pi}{3}\right)$

$x'(t) = -4\pi\sin\left(\frac{2\pi}{3}t\right)$

$y'(t) = 4\pi\cos\left(\frac{2\pi}{3}t\right)$

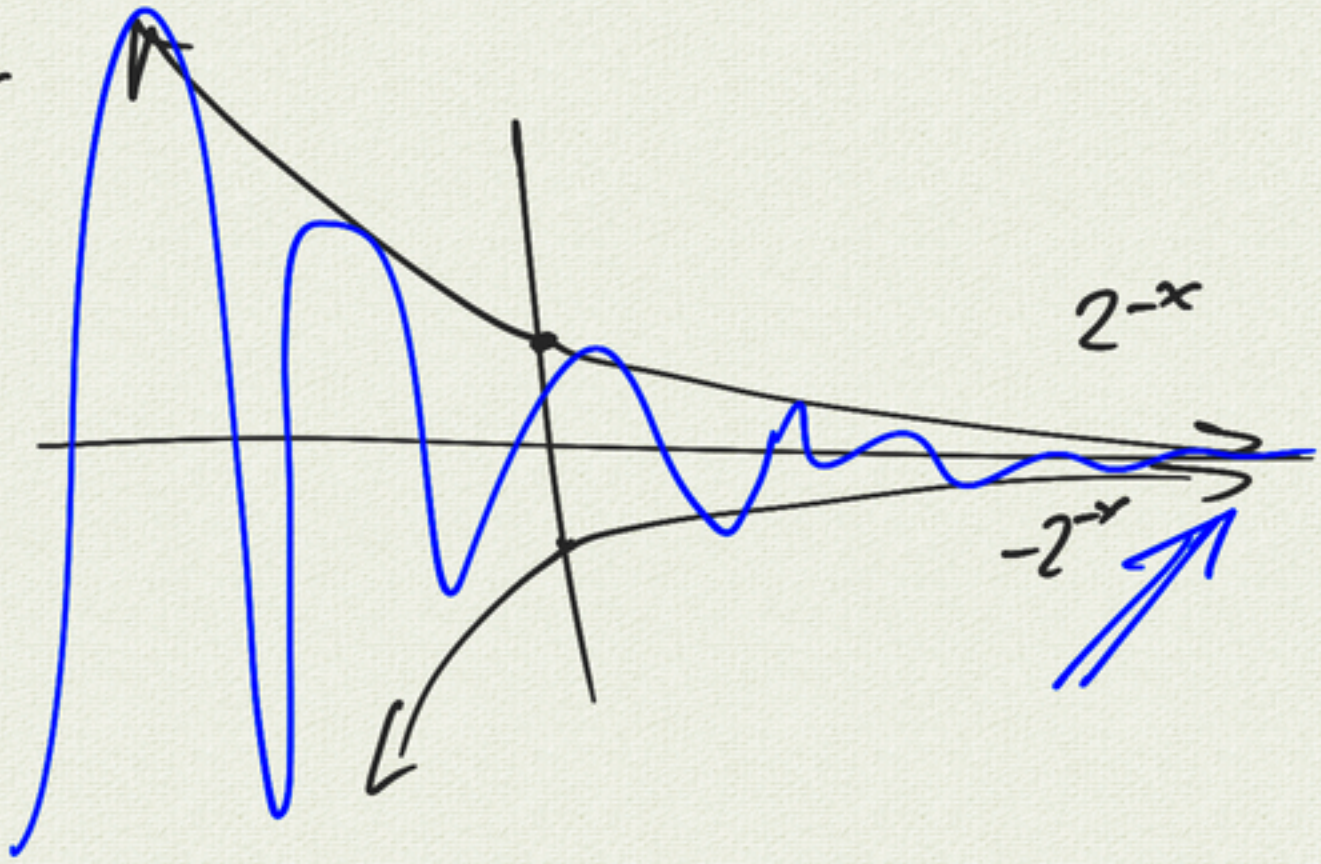


$t = 3/4$

$x'\left(\frac{3}{4}\right) = -4\pi\sin\left(\frac{\pi}{2}\right) = -4\pi$

$y'\left(\frac{3}{4}\right) = 4\pi\cos\left(\frac{\pi}{2}\right) = 0$

$$\lim_{x\to\infty} 2^{-x}\sin x = 0$$



$$P(t) = (4000)2^{t/4}$$

$$\frac{d}{dx}(2^x) = 2^x \ln 2$$

$$\Rightarrow P'(t) = 4000 \cdot 2^{t/4} \ln 2 \left(\tfrac{1}{4}\right)$$